Veto Players and Policy Development

Alexander V. Hirsch[1] and Kenneth W. Shotts[2]

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[1]California Institute of Technology, Division of the Humanities and Social Sciences. MC 228-77, Pasadena, CA 91125. Phone: (626) 395-4216. Email: avhirsch@hss.caltech.edu.

[2]Stanford GSB. 655 Knight Way, Stanford, CA 94305. Phone: (650) 725-4510. Email: kshotts@stanford.edu.

Abstract

Many political institutions use decisionmaking procedures that create *veto players*–individuals or groups who lack direct decision making authority, but nevertheless have the power to block policy change. In this paper we analyze the effect of veto players when policies are developed endogenously by actors with divergent policy goals. We first analyze policymaking activity and show that if veto players are moderate then competing groups on both sides of the political spectrum will develop policies for consideration. As veto players become more extreme, the pattern of activity becomes asymmetric, and one side disengages from policy development. For highly extreme veto players, there is no policy development and gridlock results. We also analyze the utility of centrist actors and show that veto players can have several different effects. Moderate veto players dampen productive policy competition because of their resistance to change. But some effects of veto players are surprisingly positive. In particular, when the status quo benefits a veto player and there is a skilled policy developer who is highly motivated change it, the veto player forces the developer to develop a higher quality proposal, which can be beneficial for centrists as long as the veto player isn't too extreme. We apply our model to explain changes in patterns of policy development in the U.S. Senate, and to analyze conditions under which centrist members of the Senate may choose to maintain the filibuster.

Introduction

Many political organizations use decisionmaking procedures that create *veto players*–individuals or groups who, despite lacking direct decisionmaking authority, nevertheless have the power to block policy change. For example, chief executives often have constitutionally-granted veto powers (Cameron 2000); supermajority procedures in legislatures, parliaments, and commissions generate implicit veto pivots (Brady and Volden 1997, Crombez 1996, Diermeier and Myerson 1999, Krehbiel 1998, Tsebelis 2002); and bureaucracies are sometimes structured so that an agency must seek the approval of another agency or interest group before it can act (McCubbins, Noll and Weingast 1987, Moe 1989).

Despite the ubiquity of procedures that create veto players, commentators are of two minds about their consequences. Debates over the filibuster in the U.S. Senate provide an illustration. Critics of the filibuster complain about the minority party's ability to engage in obstructionism. And from the perspective of traditional spatial models of politics, it would seem that centrist senators would clearly benefit by eliminating the filibuster. However, proponents of the filibuster have argued that hurdles to policy enactment encourage constructive deliberation (Arenberg et al. 2012).

To understand and assess the effects of veto players on policymaking it is important consider both of these possibilities, allowing for constructive policymaking as well as obstruction. We do this by extending the the competitive policy development model of Hirsch and Shotts (2015). The policy process is modeled as an open forum in which a decisionmaker relies on one or more policy-motivated actors, known as *developers*, to craft new proposals. Rather than promise policy-contingent transfers or furnish general policy-relevant information,[1] the developers gain support for their proposals by making costly, up-front policy-specific investments in their *quality*. Quality reflects characteristics of policies that are valued by all participants in the process, such as cost savings, promotion of economic growth, or efficient and effective administration. In the original model, competition benefits a unitary decisionmaker because it prevents a developer from extracting all the benefits of her quality investments in the form of ideological concessions.

In the present paper we analyze how veto players affect this process. Veto players create additional hurdles to policy change, because they have the power to block proposals that they find less desirable than the status quo. However, the effect of veto players is not obvious; additional policy hurdles may either *deter* investment in policy development, or *spur* policy developers to invest more in quality. Given these countervailing effects it is also not ex ante obvious whether, and under what circumstances, the net effect of veto players is beneficial for the policy interests of centrist decisionmakers.

We first examine how the presence of veto players affects *patterns of activity* in policy develop-

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[1]See Grossman and Helpman (2001) for a review of theories of influence.

ment. We show that veto players lead to asymmetric behavior between otherwise-symmetric policy developers, due to a non-centrist status quo. With a non-centrist status quo, the developers are differentially motivated to enact change. Specifically, when the status quo heavily favors one developer, she has little motivation to develop an alternative policy, while her opponent has a strong motivation to do so. Consequently, the *less-motivated* developer near the status quo is largely or completely inactive in policy development, while the *more-motivated* developer far from the status quo always develops a new policy for consideration. The model thus generates a natural and intuitive pattern often seen in real-world politics: the faction with the greatest interest in change actively invests to develop a credible policy alternative, while the faction that benefits from the status quo is less constructively involved in policy development.

We next examine how the *extremity* of veto players affects patterns of activity. If veto players are moderate, then competing groups on both sides of the political spectrum will develop policies for consideration. However, as veto players become more extreme, the pattern of activity becomes asymmetric, and one side disengages from policy development. And if veto players are highly extreme, there is no policy development and gridlock results.

Finally, we examine when it is in a centrist decisionmaker's interest to *maintain* the presence of veto players in the policy process. If veto players are highly moderate or highly extreme, the decisionmaker is better off eliminating them. However, if veto players are only somewhat extreme, then the decisionmaker can benefit from their presence; especially when the status quo is noncentrist. The reason is that a developer who is highly dissatisfied with the status quo is strongly motivated to develop a new policy with sufficient quality to get it enacted. When this occurs, the other developer often isn't motivated to invest in policy development because her interests are protected by the veto player who is aligned with her. Counterintuively, then, our model predicts that veto players can be highly beneficial for the decisionmaker precisely when their presence also precludes *observable* competition. Under such circumstances, the absence of observable competition reflects the strong willingness of one faction to invest in changing a lopsided status quo, rather than exogenous constraints on her competitor's ability to participate in policymaking. An important implication is that the absence of observable competition in policy development is not prima facie evidence of political dysfunction. It could instead simply reflect competing groups' differential willingness and ability to invest in changing the status quo.

As applied to the filibuster in the U.S. Senate, our model has three main implications. First, it explains why, as the Senate has become more polarized, patterns of policymaking have gone from the "textbook Congress" of the 1970s in which members on both sides of an issue generated policy options, to a pattern of highly asymmetric policymaking activity in which the majority develops policies and the minority engages in obstructionism, increasingly leading to gridlock. Second, our model provides a novel rationale for centrist Senators' support of the filibuster as an institution;

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namely that veto players who aren't too extreme can spur development of reasonably-moderate, high-quality policies. Third, our model predicts that as the Senate's veto players become increasingly extreme, centrists are likely to become increasingly willing to change institutional rules by ending the filibuster and moving to majority-rule cloture.

The paper proceeds as follows. We next summarize related literature. We then introduce the model and show how veto players affect the set of feasible policies that can be adopted. We next explain the structure of equilibrium. Then we present the main results on patterns of policy development activity and decisionmaker welfare and apply these results to the U.S. Senate. The final section concludes.

Related Literature

Our model relates to several literatures. First, a large literature considers the consequences of super-majority rules. While our analysis does not directly consider voting rules, our model can be mapped from a collective choice setting where individuals have two-dimensional preferences over ideology and quality, the individual with the median ideology acts as the decisionmaker, and supermajority rules effectively create veto "pivots" on either side (Brady and Volden 1997, Krehbiel 1998). Among the many rationales for the supermajority rules explored in the literature are policy stability (Barbera and Jackson 2004, Caplin and Nalebuff 1988), balanced budgets (Tabellini and Alesina 1990), minority protections (Aghion and Bolton 2003), insulation of the executive (Aghion, Alesina and Trebbi 2004), intergenerational conflict (Messner and Polborn 2004), information acquisition and aggregation (Persico 2004), and maximizing campaign contributions (Diermeier and Myerson 1999).

More broadly, our model relates to a literature that examines how *constraints* on a decisionmaker's discretion can improve her welfare by helping to solve dynamic inconsistency problems, such as committing to low inflation; such constraints include delegating decisionmaking (Rogoff 1985) and employing supermajority rules (Dal Bo 2006). Our analysis, in contrast, considers how constraints on a decisionmaker's discretion can improve the set of alternatives from which she selects by influencing the behavior of *other* strategic actors.

Our work also relates to previous research on veto players and blocking coalitions (Brady and Volden 1997, Crombez 1996, Krehbiel 1998, Tsebelis 2002). The vast majority of this research adopts a purely-ideological model of policy choice. In contrast, an important feature of our model is that policies have an endogenous quality dimension; there thus exists the possibility for "vote buying" by developing high-quality policies.[2] However, an important feature of our model is that quality is *policy-specific*, rather than being applicable to policies anywhere in the ideological spectrum. Thus

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[2]Anesi and Bowen (2021) analyze veto players and vote buying with transfers in a model with a binary policy that is exogenously either good or bad.

our model contrasts with the many models that build on Crawford and Sobel (1982), in which policy-relevant information is not specific to any particular policy.[3] In so doing, we build on models of policymaking by Bueno De Mesquita and Stephenson (2007), Hirsch and Shotts (2012; 2018), Lax and Cameron (2007), Londregan (2000), Ting (2011), and Hitt, Volden and Wiseman (2017). A key feature of these models is that, in contrast to the Crawford and Sobel (1982) model, an expert is able to exert informal agenda power or "real authority" (Aghion and Tirole 1997) by creating high-quality policies. Most closely related to our work is Hitt, Volden and Wiseman (2017), which briefly analyzes the case of a single developer who faces veto players. In addition to incorporating multiple potentially-active developers, our analysis differs in that we characterize effects on the moderation and quality of policies, as well whether centrists benefit from the presence of veto players.

Finally, because the cost of investing in quality is paid up-front, our model relates to previous research on all-pay contests (Baye, Kovenock and Vries 1993, Che and Gale 2003, Siegel 2009). The policy developers simultaneously make up-front payments to generate proposals with two dimensions (ideology and quality), and the decisionmaker chooses among them subject to the veto constraint. Our model has two primary differences from most previous contest models, both of which complicate the equilibrium analysis. The first difference is that, as in Hirsch and Shotts (2015), the developers in our model are policy-motivated rather than rent seeking; the loser of the contest thus cares about the exact policy that is implemented (in the terminology of Baye, Kovenock and Vries (2012) the model features a *second-order rank order spillover*). It is thus better tailored than previous contest models to political environments, where competing actors care about a collective outcome. The second difference is that our model features players without decisionmaking power who can nevertheless block policy proposals, i.e., veto players. Under such conditions, investing in quality can be strategically beneficial in multiple ways. Specifically, a developer can be motivated to produce more quality both to improve the odds that the decisionmaker prefers her policy to others, *and* to gain the support of veto players. Both of these incentives affect the equilibrium policies developed in our model.

The Model

The model takes place in three stages. First, two policy developers simultaneously craft new policies. Second, a decisionmaker chooses a policy. Finally, a pair of veto players can either approve this policy or block its enactment, in which case a status quo policy prevails.

Policy has two components: *ideology* $y \in \mathbb{R}$ and *quality* $q \in [0, \infty)$. Players' utility functions are:

$$U_i(b) = q - (x_i - y)^2$$

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[3]The Brownian motion approach developed by Callander (2008; 2011) is more similar to our model, but his model is purely spatial, whereas we model quality directly.

where x_i is i's ideological ideal point.

Policy development Each of two developers (L and R, with ideal points $x_L < 0$ and $x_R > 0$) may simultaneously invest costly resources to develop a new policy $b_i = (y_i, q_i)$ with ideology y_i and quality $q_i \geq 0$ at cost $\alpha_i q_i$. Costs are linear, and developer i's marginal cost, $\alpha_i > 2$ is sufficiently high that quality isn't a collective good between the two developers. Note that a developer will only invest in quality if it increases the probability her policy will be chosen.

Policy choice In Hirsch and Shotts (2015) and Hirsch (2022) policy is chosen by a single decisionmaker at $x_D = 0$. We augment that process with two *veto players* $x_{VL} < 0$ and $x_{VR} > 0$. If either veto player rejects the decisionmaker's chosen policy, the status quo prevails.

The set of possible policies includes the set of newly-developed policies and the *status quo* $b_0 = (y_0, q_0)$. For simplicity, we assume the status quo is low-quality ($q_0 = 0$) and is within the "gridlock interval" for 0-quality policies ($x_{VL} < y_0 < x_{VR}$).

The Effect of Veto Players on Decisionmaking

In the absence of veto players, the decisionmaker can revise any status quo to a low quality policy that exactly reflects his ideal ideology. It is thus "as if" the status quo ideology is $y_0 = x_D = 0$. The decisionmaker is then willing to adopt any policy that he prefers over $(0,0)$. This is depicted in the top panel of Figure 1; the set of acceptable policies is located above the green line that represents the decisionmaker's indifference curve through his ideal point with 0 quality.

The presence of veto players creates additional hurdles to policy change, and this affects the decisionmaking process in two ways. First, it expands the range of potential status quos that the developers may face: the status quo may be any noncentrist policy $y_0 \in [x_{VL}, x_{VR}]$. Because the status quo may not exactly reflect the decisionmaker's preferences, he will be more receptive to new proposals—this can be seen in the lower panel of Figure 1, in which the developers' policies must only be above the decisionmaker's indifference curve through $y_0 \neq 0$ to gain his support. However, for policy change to occur, the new policy *also* must be acceptable to both veto players, who are collectively more opposed to policy change than the decisionmaker. This can be seen by observing that the veto players' indifference curves through the status quo (the dashed red lines) are steeper than the decisionmaker's indifference curve. To avoid a veto, the decisionmaker must choose a policy that is above the upper envelope of these two indifference curves. We henceforth refer to this region, which is shaded in the figure, as the *veto-proof set*.

The effect of veto players hinges on how this change in the set of acceptable policies affects policy developers' strategic incentives to invest in quality. They may be less willing to invest, if they are
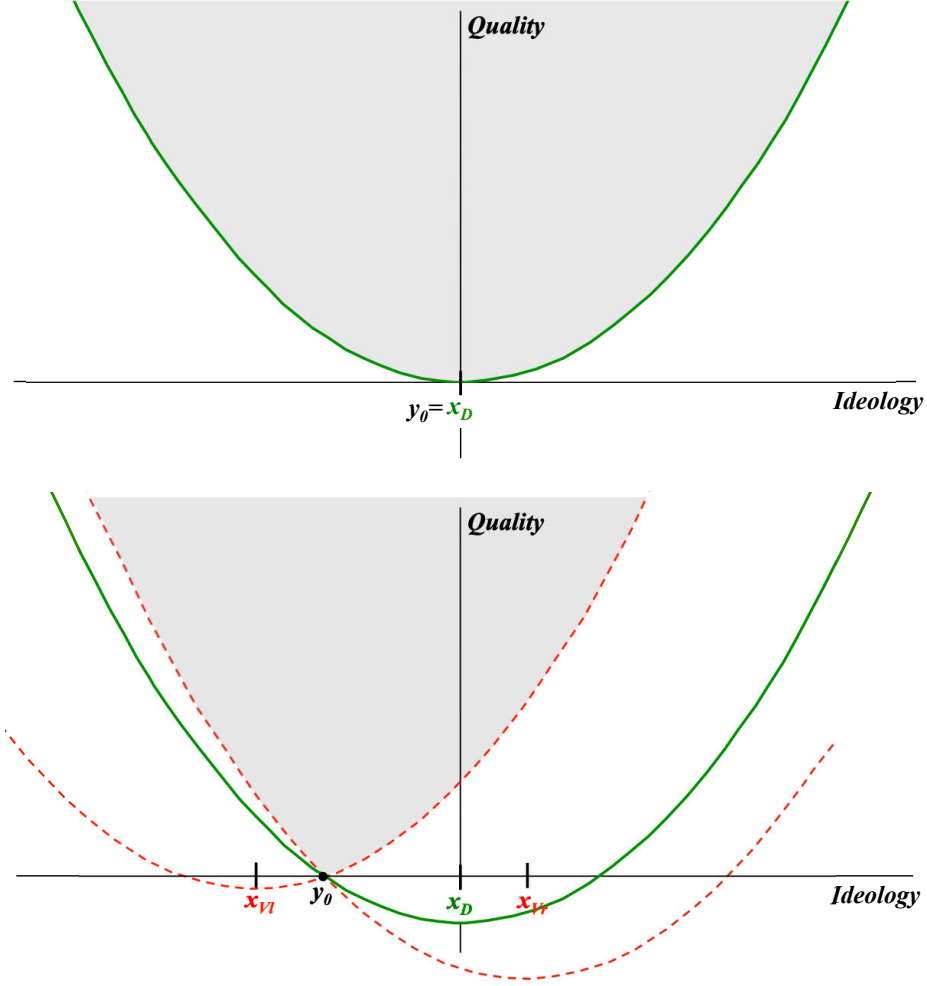
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Figure 1: *The Effect of Veto Players on Decisionmaking*

favorably disposed to the status quo or unwilling to satisfy veto players' demands. Or, they may be more willing to invest, if they strongly dislike the status quo.

Notation To characterize how veto players affect the game, we introduce additional notation and terminology. We call the decisionmaker's utility from a policy its *score*, $s(y,q) = U_D(y,q) = q - y^2$. The concept of a score is useful because it fully characterizes how the decisionmaker will evaluate veto-proof policies available to her.

Absent veto players it is "as if" the score of the status quo is $s(0,0) = 0$, and the decisionmaker chooses the policy with the highest score subject to the constraint that it is ≥ 0. Veto players increase the range of scores he is willing to accept (to those $\geq U_D(y_0,0) = -y_0^2$), but also restrict the set of acceptable policies *given each score*. The following defines the veto-proof set in terms of
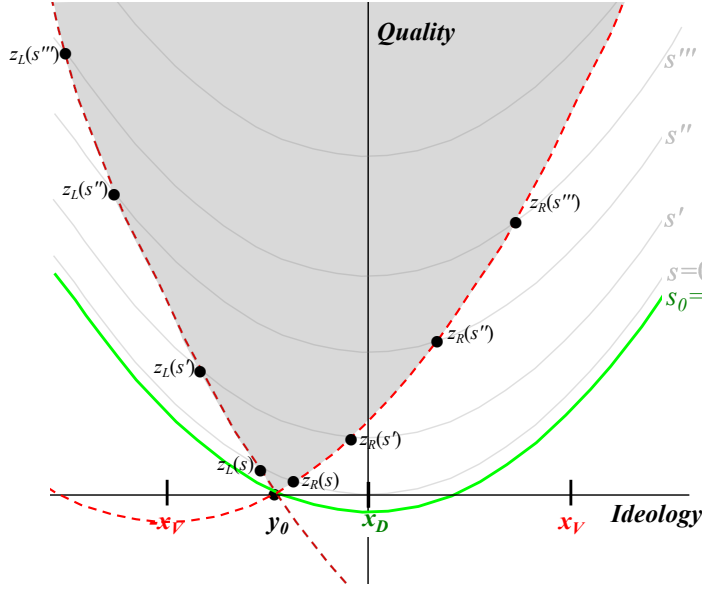
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Figure 2: *The Veto-Proof Set*

scores.

Definition 1 *Let* $z_L(s) = y_0 - \frac{s - s_0}{2|x_{VR}|}$ *and* $z_R(s) = y_0 + \frac{s - s_0}{2|x_{VL}|}$, *where* $s_0 = -y_0^2$ *is the score of the status quo. A policy* (s, y) *with score* s *and ideology* y *(and hence quality* $q = s + y^2$ *) is veto-proof iff* $y \in [z_L(s), z_R(s)]$.

Figure 2 depicts an example. The decisionmaker's indifference curves, i.e., the sets of policies with the same score, are depicted by green lines. On any given score curve s, the range of veto-proof ideologies is $[z_L(s), z_R(s)]$. The right boundary is determined by the left veto player (who opposes rightward policy change) while the left boundary is determined by the right veto player (who opposes leftward policy change). After all policies have been developed, the decisionmaker optimally chooses the highest-score veto-proof policy.

Preliminary Analysis

Monopolist's Problem

To see how veto players influence policy development, it is helpful to first consider the case of a single developer who is a "monopolist."[4] Because a policy with score s and ideology y must have quality $q = s + y^2$, the up-front cost to developer i of crafting it is $\alpha_i(s + y^2)$, and her policy utility if it is adopted

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[4]See also Hirsch and Shotts (2015; 2018), Hitt, Volden and Wiseman (2017), and Hirsch (2022).

(whether or not she herself crafted it) is $V_i(s_i, y_i) = U_i\left(y_i, s_i + y_i^2\right) = -x_i^2 + s_i + 2x_i y_i$. A monopolist's objective is then to craft a veto-proof policy (s_i, y_i) that maximizes $-\alpha_i\left(s_i + y_i^2\right) + V_i(s_i, y_i)$, which may be written as

$$\underset{\{(s_i, y_i): s_i \geq s_0, y_i \in [z_L(s_i), z_R(s_i)]\}}{\arg\max} \left\{ \underbrace{-(\alpha_i - 1)\, s_i}_{\text{score effect}} + \underbrace{2x_i y_i - \alpha_i y_i^2}_{\text{ideology effect}} \right\}. \tag{1}$$

From Equation 1 it is easy to see what a monopolist would *like* to do absent veto players—craft a policy no better for the decisionmaker than the status quo. This means setting $s_i = s_0$ (which minimizes the loss in the first term) and developing a policy with the ideology that optimally trades off ideological concessions to the decisionmaker against the cost of compensating him with higher quality (which maximizes the second term). This optimal ideology is $y_i = \frac{x_i}{\alpha_i}$, which is a convex combination of the decisionmaker's ideal point $x_D = 0$ and the monopolist's ideal point x_i, weighted by the cost of producing quality α_i. However, *veto players prevent the monopolist from doing this*, because they force her to develop a policy within the veto-proof set. This is precisely why veto players can benefit the decisionmaker–they force a developer to craft a higher-score policy if she wishes to move policy in her preferred ideological direction.

What then does a monopolist do in the presence of veto players? She develops a policy on the closer boundary of the veto-proof set ($z_L(s_L)$ for developer L or $z_R(s_R)$ for developer R). In choosing which policy to develop, she trades off the marginal benefit of moving the ideological outcome closer to her ideal point against the marginal cost of producing enough quality to get the support of the opposite-side veto player. Substituting the optimal ideology $y_i^* = z_i(s_i)$ into Equation 1, straightforward optimization characterizes her optimal policy.

Proposition 1 *When developer i is a monopolist, she crafts the policy (s_i^{M*}, y_i^{M*}), where*

$$y_i^{M*} = \begin{cases} \max\left\{ y_0, \frac{1}{\alpha_L}x_L + \left(1 - \frac{1}{\alpha_L}\right)x_{VR} \right\} & \text{for } i = L \\ \min\left\{ y_0, \frac{1}{\alpha_R}x_R + \left(1 - \frac{1}{\alpha_R}\right)x_{VL} \right\} & \text{for } i = R \end{cases}$$

and $z_i(s_i^{M*}) = y_i^{M*}$. The monopolist invests in policy development iff the status quo is farther away from her ideal point than her ideal monopoly policy: $s_i^{M*} > s_0 \iff |y_0 - x_i| > \left|y_i^{M*} - x_i\right|$.

Thus, as in Hitt, Volden and Wiseman (2017) the optimal ideology of a monopolist's policy is a convex combination of her own ideal point and the ideal point of the binding (opposite-side) veto player, weighted by the cost of producing quality. If the status quo is already closer to the developer than this policy, she develops no policy and the status quo is maintained.

When a policy is developed, its ideological location y_i^{M*} depends solely on the tradeoff at the margin between ideological gains and costs of quality. With linear costs of quality, *the optimal*

ideology does not depend on the status quo. However, its *quality does depend on the status quo*, because a status quo that is closer to the opposite-side veto player's ideal point forces the developer to generate more quality to get his assent. This generates the following result.

Corollary 1 *At any status quo y_0 where policy development occurs ($s_0 < s_i^{M*}$), the monopoly score s_i^{M*} is strictly decreasing (increasing) in y_0 when $i = R(L)$.*

Intuitively, the farther is the status quo from the monopolist, the more change she wants, and the more the decisionmaker benefits from her efforts.

Form of Equilibrium

We next describe some properties of equilibria in the main model, in which two developers compete. We say that developer i is *active* when she develops a veto-proof policy with score $s_i > s_0$ (and therefore with strictly positive quality), and she is *inactive* if she exerts no effort and "develops" the unique veto-proof policy with score s_0, i.e., the status quo. Equilibria may be in pure or mixed strategies; we begin by discussing the former.

Pure Strategy Equilibria

The form of all pure strategy equilibria is as follows.

Lemma 1 *In a pure strategy equilibrium, the developer k with the lower monopoly score is inactive, while the other developer $-k$ crafts her monopoly policy $\left(s_{-k}^{M*}, y_{-k}^{M*}\right)$ from Proposition 1.*

In a pure strategy equilibrium, at least one developer must be inactive. The reason is simple: if both were active, one of them would be strictly better off either dropping out, or producing slightly more quality and winning for sure. In addition, the inactive developer must have the lower monopoly score; otherwise, her opponent would strictly prefer to enter and win with her monopoly policy.[5] Finally, the active developer must develop her monopoly policy, because absent competition her incentives are the same as those of a monopolist's. (If both developers' monopoly scores are s_0, then each prefers not to enter the contest, and in equilibrium both remain inactive).

While the preceding explains why pure strategy equilibria must take a particular form, it does not explain why they exist at all – why doesn't the inactive developer simply craft a policy slightly better for the decisionmaker than her opponent's policy? Indeed, this is precisely what occurs in the model absent veto players, which lacks pure strategy equilibria (Hirsch (2022)). Intuitively, the reason is that the veto players force the active developer to craft a policy that is sufficiently moderate and high-quality to also *insulate it* from potential competition by the other developer.

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[5] A simple sufficient condition for this argument to hold is that $|x_i| \geq |x_{V-i}|\ \forall i$, i.e., each developer is weakly more extreme than the opposite veto player.
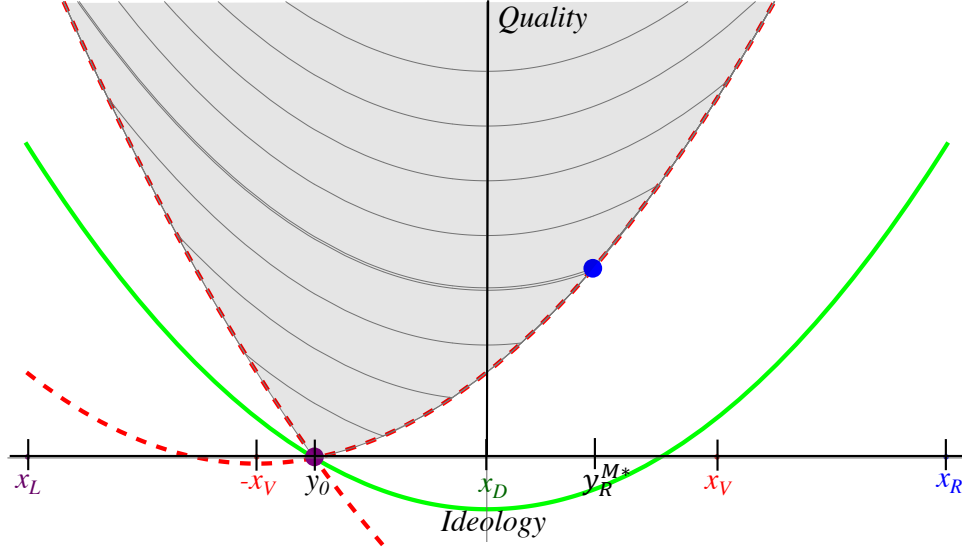
Figure 3: *A Pure Strategy Equilibrium*

Example 1: pure strategy equilibrium   Figure 3 depicts a particular set of parameter values such that there is a pure strategy equilibrium. In this example, R develops a new policy (represented by the blue dot in the figure) that is sufficiently high-quality to get the assent of the left veto player. Because the status quo is close to x_{VL}, R must make substantial quality investments to get her policy enacted. Moreover, R's policy is sufficiently high-quality that L prefers to sit out rather than developing her own competing policy. This is indicated by the purple dot at the status quo.

Mixed Strategy Equilibria

Mixed strategies in our model are potentially quite complicated. Developers could be either active or inactive, and when active they could mix over a continuum of scores, as well combinations of ideology and quality to deliver a particular score. Despite this potential complexity, we show in the Appendix that it is without loss of generality to consider strategy profiles of the following form.

Remark 1 *We restrict attention to strategy profiles in which each developer*

1. *only crafts veto-proof policies* $(s_i \geq s_0 \text{ and } y_i \in [z_L(s_i), z_R(s_i)])$

2. *chooses the score* s_i *of her policy according to a cumulative distribution function* $F_i(s_i)$

3. *crafts a unique policy* $(s_i, y_i(s))$ *at each score* s_i.

   As in most contest models, it is useful to focus on the distribution over the scores of the policies that are developed. To get her policy enacted, a developer needs to offer the decisionmaker a veto-

proof policy with a higher score than what her opponent offers. The developers mix over scores as follows.

Proposition 2 *In any mixed strategy equilibrium satisfying the conditions in Remark 1, there is a developer k and two scores $\underline{s}$ and $\overline{s}$ satisfying $s_0 \leq \underline{s} < \overline{s}$ such that*

- *developer k's score CDF F_k has support $s_0 \cup [\underline{s}, \overline{s}]$ and exactly one atom at s_0,*

- *developer $-k$'s score CDF F_{-k} has support $[\underline{s}, \overline{s}]$ and exactly one atom at $\underline{s}$.*

Mixed strategy equilibria have three properties. First, both developers mix smoothly over policies with a range of scores in a *common interval* $[\underline{s}, \bar{s}]$. Second, one developer k has an atom at s_0, meaning that she is *sometimes inactive* in the sense of not developing any new policy. Third, the other developer $-k$ has an atom at $\underline{s}$; when $\underline{s} > s_0$ (which generically is the case in equilibrium) this means she is *always active*, but develops the exact policy $(\underline{s}, y_{-k}(\underline{s}))$ with strictly positive probability.

For intuition as to why mixed strategy equilibria must take this form, we consider and rule out some other possible types of strategy profiles, focusing on the generic case of $\underline{s} > s_0$.[6] First, suppose developer i sometimes develops a policy with a score $\tilde{s}_i$ strictly greater than the highest-score policy produced by her opponent, meaning that $\tilde{s}_i$ is strictly higher than necessary to ensure enactment of i's policy. Developer i would only do this if she would *also* want to develop this policy as a monopolist; but then the strategy profile in question must involve her *only* developing this policy (and none with other scores), further implying that $-i$ must be inactive, i.e., the profile is actually in pure strategies rather than mixed strategies. A similar argument rules out gaps within the common score interval $[\underline{s}, \overline{s}]$. Another possibility is that developer i has an atom at some score $\hat{s}_i$ strictly inside the common score interval, i.e., $\hat{s}_i \in (\underline{s}, \overline{s})$. But then the policies that her opponent is developing with scores slightly below this atom cannot be optimal, because $-i$ can profitably deviate to a score just above the atom and achieve a discrete increase in the probability her policy is enacted.

Finally, it is not only possible but necessary for each developer to have an atom at *either* s_0 or $\underline{s}$. Otherwise, her opponent would be unwilling to develop policies with scores slightly above $\underline{s}$, because doing so would mean paying strictly positive costs of policy development while almost always losing. We lastly argue that exactly one developer must have an atom at s_0, and the other at $\underline{s}$. To see why, first note that at most one developer can have an atom at $\underline{s}$; if both did, one could profitably deviate to a score just above the atom and achieve a discrete increase in the probability that her policy is enacted. Next, at most one developer can have an atom at s_0; otherwise, by the preceding there would be at least one developer (say j) who has an atom at s_0 and faces an opponent *without* an atom at $\underline{s}$. Equilibrium requires that this developer be indifferent between crafting policies at

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[6]If $\underline{s} = s_0$ both developers must have atoms at s_0, but the reasons are more subtle; see Appendix for details.

s_0 and $\underline{s}$, because otherwise she would be unwilling to craft policies with scores slightly above $\underline{s}$. However, this is impossible, because any policy (s_j, y_j) with a score $s_j \in [s_0, \underline{s}]$ would have the same probability of winning, and over such a range there is a unique optimal score.

Although the form of mixed strategy equilibria is reasonably intuitive, the details are cumbersome to derive by hand. However, as described in the Appendix, equilibria can be computed numerically.

Example 2: mixed strategy equilibrium Figure 4 presents an example of a mixed strategy equilibrium. The left panel depicts score CDFs while the right panel depicts policies. In this example, developer R is always active, whereas L is inactive with probability $F_L(s_0)$. This is intuitive, because R is more dissatisfied with the status quo, which is $y_0 < 0$ in the example.

Looking at R's strategy, with probability $F_R(\underline{s})$ she develops a policy exactly at the blue dot in the right panel. Otherwise she mixes over the policies on the blue curve with scores in $(\underline{s}, \bar{s}]$. Her policies are fully constrained by the left veto player, and are on the boundary of the veto-proof set. It may seem counterintuitive that R sometimes produces a policy at score $\underline{s}$ because L (when active) never develops a score below $\underline{s}$. Thus, R could develop a cheaper, lower-score policy and still win with the same probability, $F_L(s_0)$. But R doesn't just care about the decisionmaker's support; she also needs to gain the assent of the left veto player. And just as a monopolist is willing to craft a policy at a score strictly greater than s_0 to gain the approval of the opposing veto player, so too is a developer whose opponent is sometimes inactive. In this example, R's optimal score-$\underline{s}$ policy trades off the costs of developing a policy that will gain the left veto player's assent (which she incurs with probability 1) against the benefits of getting an ideological outcome closer to her ideal point when her opponent chooses to be inactive (which happens with probability $F_L(s_0)$).

Turning now to developer L, with probability $F_L(s_0)$ she is inactive and develops no policy (the purple dot on the right panel). With the remaining probability, she mixes over policies on the purple curve with scores in $(\underline{s}, \bar{s}]$. She is willing to invest in developing these policies because they sometimes win due to the fact that R has an atom at $\underline{s}$. In this example, L's equilibrium policies are unconstrained by the veto players, i.e., they are not on the boundary of the veto-proof set.[7] Finally, the left panel shows that R's score CDF first order stochastically dominates L's score CDF, implying that the decisionmaker is strictly more likely to enact R's policy than L's.

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[7]The fact that the more-motivated developer's policies are on the boundary of the veto proof set but her opponent's are not is specific to this example, not a general property. Sometimes both are on the boundary, sometimes neither, and sometimes they transition from being on to off the boundary at scores in $(\underline{s}, \bar{s})$. At a point where a developer's scores are off the boundary, she is motivated solely by competition. On the boundary, she is motivated by the need to get the opposing veto player's assent. The only general property is that the policies of the always-active developer must *begin* on the boundary, because she must be motivated by gaining the veto players' support at $\underline{s}$ (otherwise she would prefer to deviate to scores in $(s_0, \underline{s})$ that would gain the decisionmaker's support with the same probability). The complexity of boundary conditions is why numerical analysis is necessary to solve mixed strategy equilibria.
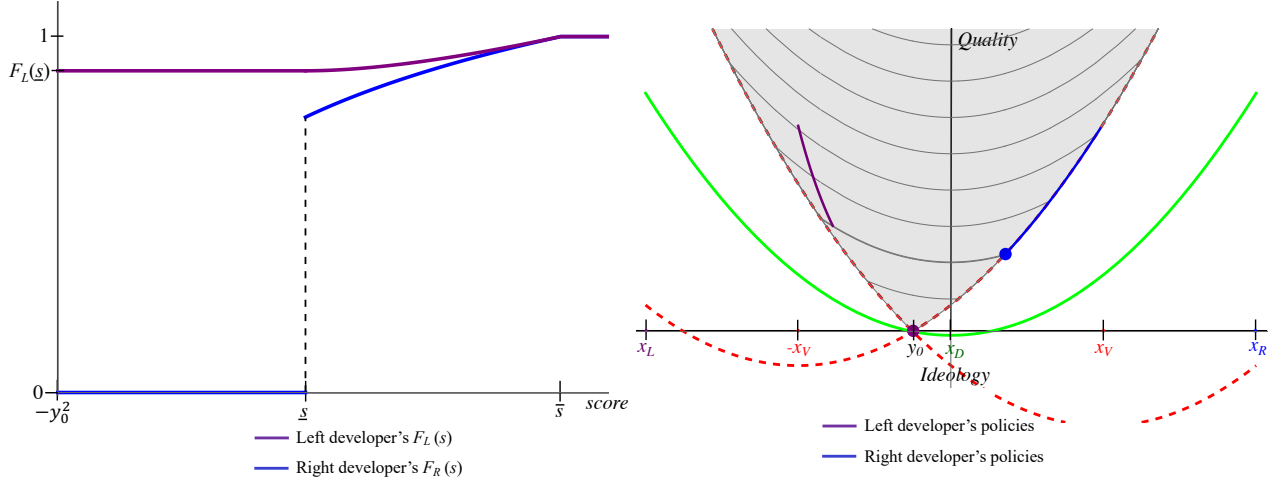
Figure 4: A Mixed Strategy Equilibrium

Main Results

We now state our main results. For simplicity we henceforth restrict attention to the case of developers who are equally capable ($\alpha_L = \alpha_R = \alpha$), and developers and veto players who are equidistant from the decisionmaker ($-x_L = x_R = x_E$, $-x_{VL} = x_{VR} = x_V$). Under these assumptions, any asymmetry in developers' incentives must arise from the location of the status quo.

We refer to the developer farther from the status quo as *more-motivated*, and her opponent as *less-motivated*. The more-motivated developer is more likely to engage in policy development for two reasons. First, she has more to gain: because ideological loss functions are common and convex, she places a greater marginal value on shifts in her direction from the status quo. Second, she has an easier time persuading the opposing veto player to consent to policy changes; for example, if the status quo is $y_0 < 0$, it is easier to get the left veto player to agree to a rightward policy shift than it is to get the right veto player to agree to a leftward policy shift.

Patterns of Activity

Patterns of activity in our model depend on incentives to engage in policy development. What *induces* a developer to be active? Shifting policy in her ideological direction; this incentive is greater when the other possible outcome (either the status quo or her opponent's policy) is far from her ideal point. On the other hand, what *deters* a developer from being active? The cost of developing a policy that can be enacted; this cost is higher when the opposing veto player is an extremist who demands lots of quality to compensate for small ideological movements, and is also higher when the opposing developer crafts a high-quality policy that is appealing to the decisionmaker. The interplay
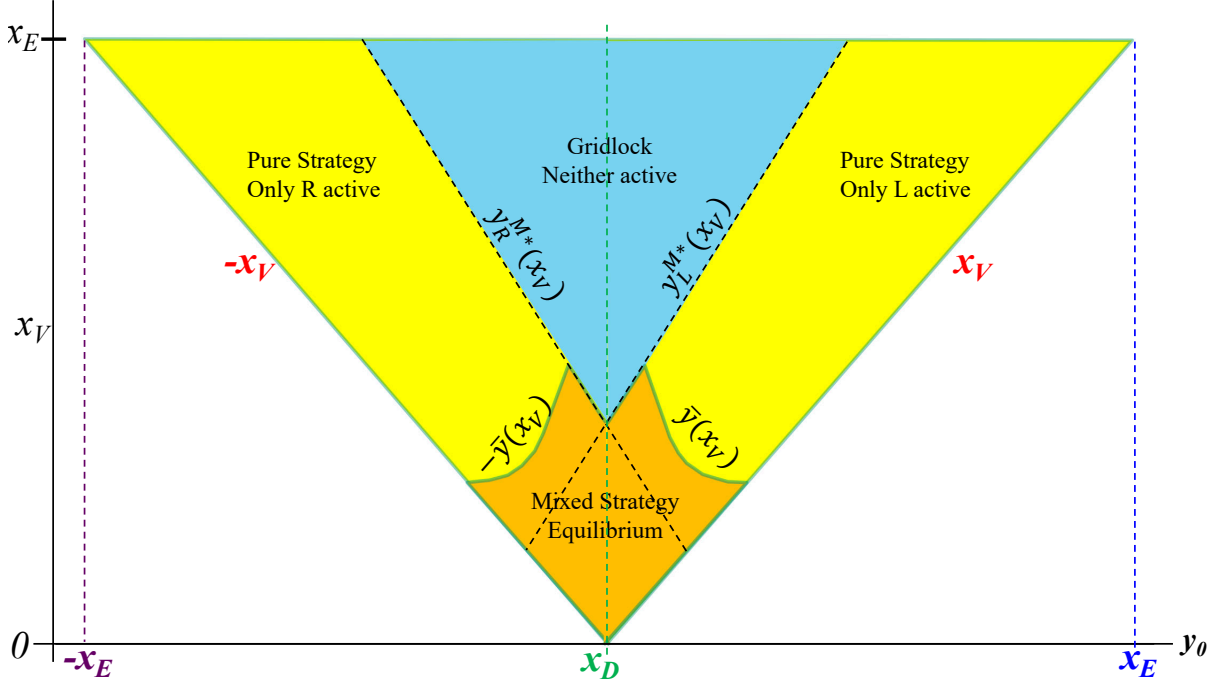
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Figure 5: *Patterns of Activity as a Function of y_0 and x_V*

between these motives generates three possible patterns of equilibrium activity: (i) neither developer is active, (ii) only the more-motivated developer is active, or (iii) the more-motivated developer is always active, the less-motivated developer is *sometimes* active, and the equilibrium is in mixed strategies. Which of these arises depends on the extremity of the veto players and the location of the status quo. Figure 5 gives an example, varying x_V (on the vertical axis, between 0 and x_E) and y_0 (on the horizontal axis, between $-x_V$ and x_V).

The first possibility (neither developer is active) occurs in the blue region of Figure 5. In this region the veto players are extreme and the status quo is moderate. Each developer chooses not to develop a policy because it is too costly to get the opposing veto player's approval. The necessary condition for this case comes from our analysis of a monopolist. Recall from Proposition 1 that a monopolist refrains from developing a policy if the status quo is closer to her ideal point than her monopoly policy $y_i^{M*}(x_V)$ (denoting the dependence on x_V explicitly). Thus, there is a pure strategy equilibrium in which neither developer is active if the status quo is both to the left of L's monopoly policy and to the right of R's monopoly policy, i.e., if it's sufficiently moderate, $y_0 \in \left[y_R^{M*}(x_V), y_L^{M*}(x_V)\right].$[8] From the definition of the monopoly policy $y_i^{M*}(x_V)$, we can also see that this requires the veto players to be sufficiently extreme, i.e. $x_V \geq \frac{x_E}{\alpha-1}$, so that $y_R^{M*}(x_V) \leq y_L^{M*}(x_V)$.

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[8]As can be seen from the definition of y_i^{M*} in Proposition 1, if the cost of policy development is arbitrarily large ($\alpha \to \infty$), our model reduces to the classic spatial model with gridlock for status quos in $(-x_V, x_V)$.

The other two possibilities occur outside of the blue region of Figure 5, i.e., for parameter values such that at least one policy developer would be active as a monopolist. Not surprisingly, the set of active developers with competition always includes the more-motivated one. The question of whether the less-motivated developer is inactive (the yellow regions), or also active with strictly positive probability (the orange region), depends on what the developer would like to do when her more-motivated competitor acts as a monopolist; will she let this policy be enacted, or step in and develop her own alternative? By Proposition 1, a monopolist's policy remains ideologically fixed at $y_i^{M*}(x_V)$ but becomes increasingly high-quality the closer the status quo is to the ideal point of the opposing veto player. This in turn makes it both more difficult, and less intrinsically beneficial, for the less-motivated developer to craft a competing policy.

Thus, when the status quo is sufficiently non-centrist in her direction, the less-motivated developer is unwilling to develop a competing policy (the yellow regions in Figure 5). In the Appendix we characterize a cutpoint $\bar{y}(x_V)$ such that R is inactive if and only if the status quo is to the right of $\bar{y}(x_V)$, and L is inactive if and only if the status quo is to the left of $-\bar{y}(x_V)$. Thus, only the more-motivated developer is active if the status quo is more extreme than this cutpoint.

Conversely, if the status quo is more moderate than $\bar{y}(x_V)$ (the orange region in Figure 5) equilibrium must sometimes involve active competition; the intuition is as follows. With moderate veto players and a moderate status quo, the more-motivated developer only needs to invest in a small amount of quality to get the opposing veto player to agree to a policy change. But if she did this, her opponent would only need to invest in a small amount of quality to swing policy back in her preferred direction. Thus, in equilibrium both developers are active (the more-motivated developer always, and the less-motivated developer with strictly positive probability), and they compete to craft policies that are both more appealing to the decisionmaker, and acceptable to the veto players. In equilibrium, the more-motivated developer's policies are more appealing for the decisionmaker than her opponent's in a first-order stochastic dominance sense.[9]

We now summarize the preceding results.

Proposition 3 *Equilibria of the model depend on the extremism of the veto players and the status quo as follows:*

1. *If $x_V \geq \frac{x_E}{\alpha-1}$ and $y_0 \in \left[y_R^{M*}(x_V), y_L^{M*}(x_V)\right]$, neither developer is active.*

2. *Otherwise, at least one developer is active:*

   (a) *The more-motivated developer is active with probability 1.*

   (b) *If $y_0 \notin [-\bar{y}(x_V), \bar{y}(x_V)]$, the less-motivated developer is never active.*

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[9]This is also trivially true for pure strategy equilibria in which only the more-motivated developer is active.

(c) If $y_0 \in [-\bar{y}(x_V), \bar{y}(x_V)]$ there is a mixed strategy equilibrium in which the less-motivated developer is sometimes active.

3. The more-motivated developer's policies have FOSD higher scores, and thus are more likely to be enacted than the less-motivated developer's policies.

At a broad level, the proposition shows that *asymmetric activity* is a fundamental feature of our model – even when the developers are symmetrically extreme and equally capable. Whenever there is activity, the more-motivated developer is always active, and her opponent is either completely inactive, or mixes between being active and inactive.[10]

The effect of competition A natural question to ask is how the presence of each developer affects her opponent's decision about whether to engage in policy development. In one direction this question is trivial: the more-motivated developer's willingness to develop an alternative to the status quo is *unaffected* by the presence or absence of a competing developer on the opposite side of the ideological spectrum. That is, for all parameter values, she is active in the presence of competition if and only if she would be active as a monopolist.[11]

In contrast, the less-motivated developer's willingness to develop a new policy *is* affected by the presence of a more-motivated competitor. Moreover, a competitor's presence can either *increase* or *decrease* her policy development activity; this can be seen by considering different sub-regions of the mixed-strategy regions within which the less-motivated developer is sometimes active. Recall that the dashed lines within this region of Figure 5 depict $y_L^{M*}(x_V)$ and $y_R^{M*}(x_V)$, the ideological locations of each developer's optimal policies as a monopolist. Thus, when the status quo is within the orange region but outside $\left[y_L^{M*}(x_V), y_R^{M*}(x_V)\right]$, the less-motivated developer *would not* be active absent a competitor, but is active with her. The reason is that absent competition the status quo is insufficiently distasteful to motivate costly policy development, but the presence of a more-motivated competitor attempting to pull policy *even further* in her direction is. Conversely, when the status quo is both within the orange region and within $\left(y_L^{M*}(x_V), y_R^{M*}(x_V)\right)$, the less-motivated developer would *always* be active at policy development absent a competitor, but is sometimes *inactive* with one. The reason is that the presence of a more-motivated developer who is crafting an ideologically distant but also high-quality policy sometimes deters her from developing an alternative (i.e., she is inactive with strictly positive probability). Notably, this pattern of activity contrasts starkly with the model of counteractive informational lobbying in Austen-Smith and Wright (1994), in which one interest group's engagement can only *motivate* activity by opposing interests.

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[10]The special case $y_0 = 0$ has symmetry in developers' activity.

[11]Of course, in a mixed strategy equilibrium the policies she develops differ from her monopoly policy.

Effect of veto players' ideological extremism   We next examine how the ideological extremism of the veto players affects patterns of policy development competition. As can be seen toward the bottom of Figure 5, if veto players and the status quo are both moderate, the more-motivated developer is always active, and her opponent is sometimes active. As x_V increases (moving vertically in the figure), the probability that the less-motivated developer is active decreases monotonically. For sufficiently high values of x_V, even the more-motivated developer may be deterred from developing an enactable policy. Formally, we have the following result.

Proposition 4 *The extremism of the veto players affects policy development activity as follows.*

1. *The probability that the less-motivated developer is active is strictly decreasing in x_V, unless the equilibrium is in pure strategies, in which case it is constant at* 0.

2. *The more-motivated developer is active if and only if the veto players are sufficiently moderate,* $x_V < \frac{\alpha|y_0|+x_E}{\alpha-1}$.

Thus, increasingly extreme veto players reduce the total amount of participation in policy development. At lower levels of extremism, they make policy-development activity more asymmetric; the less-motivated developer increasingly disengages from developing policies, while the more motivated developer continues to participate. At higher levels of extremism they deter the more motivated developer from development as well—the result is gridlock and legislative stalemate.

Changes in Senate policymaking   As a brief empirical application, we argue that our model's predictions are broadly consistent with patterns of policy development in the U.S. Senate over the past few decades. As is well-established in the literature, the Senate has become increasingly polarized. In the context of our model, what matters is the extremism of the veto players; specifically, the difference between the ideal points of the 40th and 61st most liberal senators, which has increased since the 1970s. Our model predicts that increasingly extreme veto players lead to asymmetric policy development activity and ultimately to stalemate. Both of these patterns are well-documented in the empirical literature.

The first pattern – asymmetric activity – can be seen by contrasting the current highly-partisan policy development process against the traditional "textbook Congress," in which members of both parties actively worked in committees to develop proposals that could be enacted. Over time, Senate majority party leaders have played an increasingly central role in "negotiating the details of major bills" (Smith 2011, p. 135) and "shaping the content of legislation" (Smith and Gamm 2020, p. 216). For their part, members of the Senate minority have disengaged from creating policy proposals that might be enacted, and have instead adopted a strategy of obstructionism, trying to block passage of the majority's legislation (Lee 2016).

The second pattern – stalemate – is also well-established in the literature; it has become difficult for anyone, including majority party leaders, to get substantial new policies enacted. Nowadays, major policy change typically occurs via budget reconciliation (which doesn't require supermajorities) or at times of extraordinary crisis, such as 9/11, the financial meltdown of 2007-8, and Covid-19. For most policy issues (including salient ones), legislative gridlock and stalemate have become common (Binder 2015).

Thus, both the increasing asymmetry in policy development activity and the overall decline in successful policy development are consistent with our model, which provides an explanation for what many scholars see as the decline of the Senate as an effective institution for crafting public policy. As noted by (Smith 2014, p. 14): "An institution that once encouraged creativity, cross-party collaboration, individual expression, and the incubation of new policy ideas has become gridlocked."

Decisionmaker Welfare

We conclude our analysis by considering how the presence of veto players affects the welfare of centrist legislators. Specifically, we consider under what conditions the decisionmaker benefits from eliminating the veto players and allowing policy development to proceed in their absence.

In classic spatial models without the possibility for policy development, the decisionmaker *always* benefits from eliminating veto players; doing so enables him to revise a non-centrist status quo $y_0 \neq 0$ to reflect his own ideal point. With the potential for policy development, however, veto players don't always induce gridlock, because policy can be changed if one or both developers craft policies that the veto players prefer to the status quo. This opens up the possibility that veto players may benefit the decisionmaker because of how they affect the actions of the policy developers.

Welfare without veto players   To conduct our analysis we first establish a baseline for decisionmaker utility in the absence of veto players. Crucially, the relevant baseline is *not* the decisionmaker's utility for a zero-quality policy at his ideal point, as it would be in a classic spatial model. Rather, it is her expected utility from *competitive policy development in the absence of veto players.* To calculate this utility we use the Hirsch and Shotts (2015) analysis of the model without veto players, in which the two developers are always active in equilibrium and mix over policies with strictly positive scores.

Corollary 2 *Absent veto players, the decisionmaker's utility is* $EU_D^0 = x_E^2 \int_0^1 \left(\int_0^p \frac{8qp}{\alpha(\alpha-q)} dq \right) dp.$

In the model without veto players, the decisionmaker's expected utility does not depend on the location of the status quo; even if the status quo is initially non-centrist, once the veto players are eliminated it is "as if' the status quo is the decisionmaker's ideal point with 0 quality.[12] Also, note

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[12]This property holds even if decisionmaker's choice is restricted to the developers' policies and the status quo,
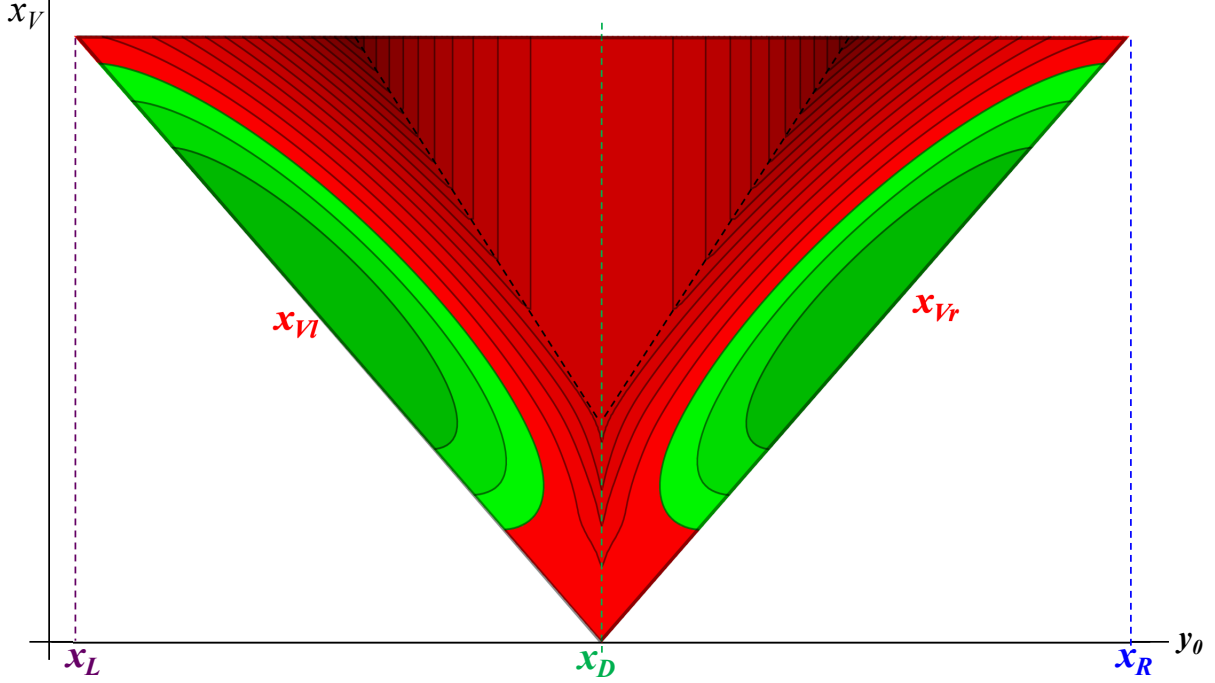
Figure 6: *Net Utility Gain from Eliminating Veto Players as Function of y_0 and x_V*

that $EU_D^0 > 0$ (because $\alpha > 1$) – reflecting the fact that a unitary decisionmaker strictly benefits from competitive policy development relative to receiving his own ideal point with 0 quality – and that the magnitude of these benefits depends on the marginal cost α of policy development.

Welfare with veto players   Having established the relevant baseline we next analyze the decisionmaker's expected utility in the presence of veto players using Proposition 3; we denote this as $EU_D^{VP}(x_V, y_0)$. When there is gridlock, the decisionmaker's utility is $EU_D^{VP}(x_V, y_0) = -y_0^2$, which is unambiguously worse than his utility from competitive policy development absent veto players. When there is a pure strategy equilibrium with one active developer, $EU_D^{VP}(x_V, y_0)$ is the score of that developer's monopoly policy from Proposition 1. Finally, when there is a mixed strategy equilibrium, we calculate $EU_D^{VP}(x_V, y_0)$ using numerical integration. Comparing $EU_D^{VP}(x_V, y_0)$ against EU_D^0 yields the following result.

Proposition 5 *The decisionmaker strictly prefers to eliminate the veto players $\left(EU_D^0 > EU_D^{VP}(x_V, y_0)\right)$ if the veto players are sufficiently moderate or the status quo is sufficiently moderate. Otherwise he strictly prefers to maintain them.*

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because the more-motivated developer can profitably defeat the status quo by offering the decisionmaker his ideal point with 0 quality.

Figure 6 illustrates when the decisionmaker would be better off eliminating the veto players as a function of the veto players' extremism (on the vertical axis) and the location of the status quo (on the horizontal axis). In the red region the decisionmaker benefits from eliminating veto players; clearly, this must encompass the region within which the veto players induce gridlock (the inner triangle). Conversely, in the green region he benefits from preserving them. The figure has three important features. First, there *is* a green region – in contrast to a classic spatial model, the decisionmaker *can* sometimes benefit from the presence of veto players. Second, a necessary condition for the decisionmaker to benefit is that the status quo is *noncentrist* – this contrasts with a classic spatial model, in which the worst status quos from a centrist's perspective are ones that are gridlocked at a point far from his ideal point. Third, *observable* competition isn't necessary for the decisionmaker to benefit from the presence of veto players. In fact, within most of the green region in Figure 6 (where the decisionmaker benefits from the veto players) only the more-motivated developer is active, as can be seen by comparison with Figure 5.

Why can a centrist decisionmaker benefit from the presence of veto players when the status quo is noncentrist and only one developer is active? The crucial observation is that a developer is *most willing* to invest in quality to change policy when she strongly dislikes the status quo, i.e., when it is far from her ideal point. It is thus under these circumstances that a somewhat-extreme opposing veto player can benefit the decisionmaker by credibly demanding higher quality to consent to policy change. Why then does this coincide with reduced participation in policy development? Because by forcing the developer to craft a higher-quality policy to gain her support (Corollary 1), a more extreme veto player inadvertently causes her to *deter* the less-motivated developer from crafting an alternative.[13] Overall, the surprising empirical implication is that in competitive political environments with veto players, the absence of direct competition—and apparent monopoly over policy development by one side—is not necessarily indicative of dysfunctional politics. Rather, this pattern can occur when there is an extreme status quo on a policy issue that only one party or faction is highly motivated to change. Under these conditions, the veto player who is aligned with the status quo already extracts substantial quality investments from the more motivated side, so potential competing groups rationally calculate that they are better off remaining inactive.

Having discussed when and why the decisionmaker can benefit from the presence of veto players, we now discuss what can go wrong, i.e., what happens in the red region of Figure 6 where the presence of veto players harms the decisionmaker. Veto players can have three distinct negative effects: (i) dampening productive competition, (ii) inducing gridlock, and (iii) allowing for new policies that

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[13]The intuition is similar for parameter values at which both developers are active with strictly positive probability and the decisionmaker benefits from veto players (i.e., the lowest part of the green regions in Figure 6, which overlaps with the orange mixed strategy region in Figure 5). In this case, the more-motivated developer's policies are sufficiently high quality to *sometimes*, but not always, deter the less-motivated developer from participation.

are both non-centrist and relatively low quality.

The first effect occurs when both the veto players and the status quo are very moderate, as in the bottom center of Figure 6. In this region, policy change is easy to achieve because veto players are moderate, but the developers aren't highly motivated to invest in quality because the status quo is also moderate. Correspondingly, while the equilibrium is in mixed strategies (with both developers sometimes active, see Figure 5), the presence of veto players simply dampens the intensity of productive competition. Specifically, from each developer's perspective, veto players limit both the upside of engaging in development (by constraining policy change in her own direction) and the downside of disengaging from development (by constraining policy change in her opponent's direction).

The second effect occurs when the veto players are more extreme, but the status quo is still moderate. In this case, the veto players demand a lot of quality to consent to policy change, but a moderate status quo limits the developers' motivation to provide this quality. The result is gridlock, with both developers declining to craft a new policy (see the triangular region in the top center of Figure 6, which corresponds to the blue triangle in Figure 5). Importantly, veto-player induced gridlock in our model is *worse for the decisionmaker* than in the classic spatial model. In our model, gridlock does not simply prevent the decisionmaker from changing policy to reflect his ideal; it also shuts down productive policy competition.[14]

The third effect occurs when the veto players are more extreme, and the status quo is *neither* sufficiently moderate to induce gridlock, nor sufficiently extreme to adequately motivate the more-distant developer. This effect dominates in the portions of the red region in Figure 6 for which only the most-distant developer is active (i.e., the overlap with the yellow regions of Figure 5). In these regions the active developer crafts a non-centrist policy of sufficiently quality to gain the veto players' support over the status quo, but of insufficient quality to overcome the benefits the decisionmaker would enjoy absent veto players in the form of greater centrism and more active policy competition.

Overall, our analysis of welfare shows that veto players can, under certain circumstances, be beneficial for centrists. However, this effect depends both on location of the status quo and on how extreme the veto players are in complex and nuanced ways.

Very extreme veto players We last consider a special case that is particularly relevant in an era of political polarization; veto players who are very extreme ($x_V = x_E$). Under these conditions the equilibrium is always in pure strategies, with either gridlock or a single active developer. The effect of extreme veto players on the decisionmaker's welfare depends crucially on whether the costs of policy development are greater than a cutpoint $\tilde{\alpha} \approx 3.68$ that we derive in the Appendix.

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[14]The decisionmaker's utility loss due to veto players is $EU_D^0 + y_0^2$ as opposed to y_0^2 in a classic spatial model.

Proposition 6 *With extreme veto players ($x_V = x_E$),*

- *If the cost of policy development is low, $\alpha \in (2, \tilde{\alpha})$, there is a cutpoint $\tilde{y}(\alpha)$ such that the decisionmaker benefits from eliminating veto players if and only if the status quo is sufficiently moderate, i.e., $y_0 \in (-\tilde{y}(\alpha), \tilde{y}(\alpha))$.*

- *If the cost of policy development is high, $\alpha > \tilde{\alpha}$, the decisionmaker benefits from eliminating veto players regardless of the location of the status quo.*

Figure 6, in which $\alpha = 3.75$, illustrates the second part of this proposition for extreme x_V (at the top of the figure) where $EU_D^{VP}(x_V, y_0) < EU_D^0, \forall y_0 \in [-x_V, x_V]$. Regardless of whether the outcome is gridlock or a policy crafted by a single active developer, the decisionmaker would unambigously benefit from eliminating extreme veto players because policy development is sufficiently costly.

Filibusters   We last use our model to reexamine a critical question in the literature on legislative organization: why does the U.S. Senate allow a submajority of 41 members to block legislation that a majority prefers over the status quo? The United States Constitution dictates that the Senate is a self-organizing body; as such, majorities aren't helplessly stuck with the filibuster as an institution. Indeed, both constitutional scholarship and Senate history support the proposition that a simple majority may, through various procedures, eliminate or modify the filibuster (Gold and Gupta 2004). But, as documented by Binder and Smith (2001), there has there never been a Senate majority in support of eliminating the filibuster on legislation by reducing the cloture requirement to 51 votes. Most recently, in early 2022 the Senate voted 52-48 against a one-time exception to the filibuster that would have made it possible to pass a voting rights bill. At the time, 21 Democratic Senators supported eliminating the filibuster, 27 supported changes such as requiring a "talking filibuster," and two of the most moderate Democrats (Joe Manchin and Kyrsten Sinema) opposed any changes (Rieger and Adrian 2022).

From the perspective of simple spatial models of policymaking, centrist Senators' support for the filibuster presents a puzzle. In such models, supermajority rules harm centrists by preventing them from altering policies to reflect their own ideal point. One explanation that has been offered is that centrists use supermajority requirements to counterbalance the power of non-centrists who have formal agenda-setting power (Krehbiel and Krehbiel 2023, Peress 2009). However, it is unclear whether arguments that assume formal agenda setting power apply to the U.S. Senate, where the absence of germaneness requirements gives individual members considerable power to ensure that their proposals are on the agenda. Indeed, party leaders in the Senate expend extraordinary effort to accommodate the scheduling demands of individual members (Oleszek et al. 2015). In addition, to the extent that formal agenda setting power exists in the Senate, this power can only exist with the consent of a Senate majority (Krehbiel 1992). Thus, any theoretical explanation of the filibuster

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that is grounded in an assumption of formal agenda setting power must also answer the question of why centrists would choose to layer on an *additional* procedure to address the shortcomings of formal agenda power, rather than simply revoke that agenda power.

In contrast, our model shows that *even in the absence of formal agenda setting power*, centrist Senators can sometimes benefit from maintaining supermajority requirements that create de facto veto players, because policy developers' need to satisfy those veto players can result in moderate and high-quality policy outcomes. As shown in Proposition 5 and Figure 6, centrists are most likely to benefit from the filibuster when the veto players it induces (the 41st and 60th most liberal Senators) are somewhat non-centrist and the status quo is also non-centrist. A non-centrist status quo could occur in policy areas that are rapidly changing, such as financial regulation or health care. And if Congress can only tackle a limited range of issues in a given session (due to limits on time and attention), the issues on which there is legislative activity are likely to be those on which the status quo is non-centrist.

However, our model does not imply that centrists *always* benefit from the filibuster. As shown in Proposition 5 and Figure 6, there are also several situations in which the presence of veto players leads to outcomes that are bad for centrists. In the context of current debates about the filibuster, one of these situations is particularly relevant; when both veto players are extreme and policy development is costly. Do these conditions hold in the contemporary Senate? The evidence suggests that the veto players induced by the filibuster, i.e., the 40th and 61st most liberal Senators, have indeed become increasingly polarized over the past few decades. Simultaneously, Congress has disinvested in its own capacity for policymaking—despite the fact that policy issues have become vastly more complicated—by substantially reducing the number of staffers, allowing personnel funding to remain constant or decrease in real terms, and reducing funding for agencies like the CBO, CRS, and GAO (Reynolds 2020). Given these changes, scholars and commentators have become concerned that it is increasingly difficult for members' offices to craft high-quality policies. In our model, if policy development is difficult and costly, then centrists are unambiguously harmed by the presence of extreme veto players and would be better off removing them (Proposition 6). Thus, our model suggests that although centrists may have benefitted from the filibuster in the past, calls for reform may therefore become increasingly persuasive if these trends persist.

Conclusion

In this paper we have explored developed a model of costly production of policy proposals in political environments where actors have divergent objectives, but also have a shared interest in the quality of policies that are enacted. In such environments, policy developers have opportunities to obtain informal agenda power by crafting policies that are well-designed but that also promote their own

objectives. Our goal has been to assess how the presence of veto players affects the nature of policies that are enacted as well as the utility of centrist decisionmakers.

Absent veto players, competing developers will always craft policies that benefit a centrist decisionmaker irrespective of the status quo policy. However, the effect of including veto players in decisionmaking depends on the location of the status quo. If the status quo is quite moderate, the dominant effect will be to dampen productive competition, thereby making the decisionmaker worse off. However, if the status quo is sufficiently noncentrist, then one of the developers will be willing to work hard to craft a high-quality alternative; an opposing veto player will then force her to do so, making the decisionmaker better off. As a byproduct of this effect, however, the other developer will also choose to remain inactive. Thus, veto players will be most beneficial when their presence also precludes observable policy competition.

A surprising implication of our analysis is that veto players are most beneficial for a centrist decisionmaker under precisely the circumstances that standard spatial models predict they are most harmful; when the status quo is non-centrist. Under such circumstances, only the faction most dissatisfied with the status quo will actively develop a new policy . However, this lack of observable competition is simply a symptom of the fact that veto players have already forced the policy developer to craft a reasonably-moderate and high quality policy. Our model thus contrasts sharply with simple spatial models of policy choice by providing a quality-based rationale for fragmented decisionmaking authority. This effect may contribute to the stability of supermajority requirements in the U.S. Senate and other political institutions that choose to maintain implicit veto rights.

Our model also yields testable predictions on the number of well-developed policy proposals that will be created for a given issue; multiple serious proposals are likely to be developed when the status quo is centrist or when veto players are absent. It further yields predictions about the quality of policies that are adopted. Quality is difficult to measure empirically because it comes from a variety of characteristics. However, if measurement issues can be overcome, one could test the model's prediction that centrist policies that are successfully enacted tend to be of mediocre quality relative to noncentrist ones, since the latter must be more carefully crafted to gain broad approval.

Finally, our model has surprising implications for institutional design of policymaking capacity in Congress. A natural intuition is that the best way to allocate policymaking capacity in polarized times is to invest in shared resources that can be used by all members for policy development. Our model suggests this intuition may be off-target, and that reformers might instead do better by giving resources to non-centrist policy developers. A natural fear is that such policy developers will inevitably use this capacity to further their own extreme objectives, as suggested by the pejoriative characterization in Drutman and LaPira (2020) of the current capacity regime in Congress as "adversarial clientilism." However, if policy developers are constrained–either by the possibility of active competition or by the presence of opposing veto players – then they will need to focus their

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energies on generating policies that are relatively high-quality and moderate in the hopes of getting them enacted.

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Supporting Information for
Veto Players and Policy Development

NOTE: THIS APPENDIX IS VERY PRELIMINARY AND VERY INCOMPLETE – DO NOT DISTRIBUTE!!!

A   Monopoly Model

Proof of Proposition 1

Without loss of generality, we consider the case $x_{bVE} \leq 0 < x_E$. We prove the proposition in three steps.

Step 1. We show that the entrepreneur either declines to develop a policy or develops one on the boundary of the veto-proof set between the entrepreneur and the status quo, i.e., $y_E \in [y_0, x_E]$ with q_E such that $y_E = z_R(s)$. Note that the entrepreneur never develops a policy with $q_E > 0$ that is outside the veto-proof set, because doing so means incurring cost $\alpha_E q_E$ and receiving no benefit. Also, note that within the veto-proof set, only policies with $y_E \in \{z_L(s), z_R(s)\}$, can be optimal to develop. If $y_E \in (z_L(s), z_R(s))$, then for sufficiently small ϵ the entrepreneur can develop $(y_E, q_E - \epsilon)$, which will be enacted and yield $(\alpha_E - 1)\epsilon$ higher utility for the entrepreneur. Finally, note that if the entrepreneur's policy is veto-proof and $y_E < y_0$ then the entrepreneur is strictly better off developing (y_0, q_E) and if $y_E > x_E$ then the entrepreneur is strictly better off developing (x_E, q_E). For $y_E \in [y_0, x_E]$ the binding veto player is to the left of y_0 so $y_E = z_R(s)$.

Step 2. We find the entrepreneur's utility from developing $y_E \in [y_0, x_E]$ with q_E such that $y_E = z_R(s)$. For such a policy, indifference of the left veto player means $q_E = (y_E - x_{bVE})^2 - (y_0 - x_{bVE})^2 = 2x_{bVE}(y_0 - y_E) + y_E^2 - y_0^2$, so the entrepreneur's utility is

$$- (x_E - y_E)^2 - (\alpha_E - 1)\left[2x_{bVE}(y_0 - y_E) + y_E^2 - y_0^2\right].$$

Taking the derivative with respect to y_E yields

$$2x_E - 2y_E - (\alpha_E - 1)(-2x_{bVE} + 2y_E)$$

which equals zero at

$$\hat{y}_E^* = \frac{1}{\alpha_E}x_E + \left(1 - \frac{1}{\alpha_E}\right)x_{bVE}.$$

For $y_0 < \hat{y}_E^*$, this weighted midpoint is optimal, whereas for $y_0 > \hat{y}_E^*$ the entrepreneur's utility is strictly higher from sitting out than it is for developing any $y_E \in (y_0, x_E]$ on the boundary of the veto-proof set.

Step 3. The decisionmaker's utility is $-(y_E^*)^2+2x_{bVE}(y_0-y_E^*)+(y_E^*)^2-y_0^2 = s_0+2|x_{bVE}(y_E^*-y_0)|$ and quality q_E^* follows directly from the definition of score.∎

B   Competitive Model with Veto Players

Two entrepreneurs (*L*eft and *R*ight) develop policies for consideration by a decisionmaker (*D*M) and a collection of veto players. A policy $b = (y,q)$ consists of an *ideology* $y \in \mathbb{R}$ and a level of *quality* $q \in [0,\infty) = \mathbb{R}^+$, which must be produced at an up-front cost. All players care about the characteristics of the exact policy ultimately chosen, rather than whose policy is chosen. Utility over policies takes the form

$$U_i(b) = q - (x_i - y)^2,$$

where x_i denotes player i's ideological ideal point. The game proceeds as follows.

Policy Development   In the *policy development* stage, each entrepreneur simultaneously chooses to invest costly resources to develop a new policy $b_i = (y_i, q_i) \in \mathbb{B}$ with ideology y_i and quality $q_i \geq 0$. The marginal cost to entrepreneur i of developing quality is α_i,with $\alpha_i > 1$.

Policy Choice   In the *policy choice* stage, the organization either chooses a policy from the set of newly-developed policies $\mathbf{b} \in \mathbb{B}^N$, or retains a *status quo policy* $b_0 = (y_0, q_0)$. For simplicity we assume that the status quo is of low quality ($q_0 = 0$).

In the original model of competitive entrepreneurship in Hirsch and Shotts (2015), policy is chosen by a single decisionmaker. In the present model we augment this decisionmaking process with $j \in K$ *veto players* with ideal points denoted x_{Vj}. Specifically, a decisionmaker with ideal ideology x_D first makes a take-it-or-leave-it proposal from the set of available policies. Then, if any veto player rejects the proposal, the status quo policy b_0 prevails. We use x_{Vl} and x_{Vr} to denote the ideal ideologies of the leftmost and rightmost veto players.

Assumptions   We make the following simplifying assumptions. First, the decisionmaker's ideal point is normalized to $x_D = 0$. Second, the two entrepreneurs are located on either side ($x_L < 0 < x_R$). Third, the leftmost and rightmost veto players are also located on either side ($x_{Vl} < 0 < x_{Vr}$). Finally, the status quo b_0 is not Pareto-dominated among the veto players by any 0-quality policy; combined with $q_0 = 0$ this implies that $y_0 \in [x_{Vl}, x_{Vr}]$.

Preliminary Analysis

We call the utility *gain* $U_D(y,q) - U_D(y_0, q_0) = q - y^2 + y_0^2$ that a policy gives the decisionmaker relative to the status quo its *score* $s(y,q)$ (see also Hirsch and Shotts (2015)). We first reparameterize

policies (y, q) to be expressed in terms of score and ideology (s, y), so the implied quality of a policy (s, y) is $q = \left(s - y_0^2\right) + y^2$. The score of the status quo is 0.

Definition 2

    1. *Player i's utility for policy (s, y) is*

$$V_i(s, y) = U_i\left(y, s + y^2 - y_0^2\right) = -x_i^2 + \left(s - y_0^2\right) + 2x_i y$$

    2. *Proposer i's cost to develop policy (s, y) is $\alpha_i\left(s - y_0^2 + y^2\right)$*

We now characterize equilibrium outcomes of the subgame commencing with the decisionmaker's proposal, which is subject to the approval of the veto players. When the decisionmaker proposes some policy (s, y), the veto players will evaluate it against the status quo $(0, y_0)$. Player i's utility difference between the two policies is

$$V_i(s, y) - V_i(0, y_0) = s + 2x_i(y - y_0),$$

which satisfies a single crossing property in x_i. Hence, if $y < y_0$, then a necessary and sufficient condition for all veto players to weakly prefer the proposal to the status quo is that the *rightmost* veto player x_{Vr} weakly prefers it, i.e., $V_{Vr}(s, y) - V_{Vr}(0, y_0) \geq 0 \iff y \geq y_0 - \frac{s}{2|x_{Vr}|} = z_L(s)$. Similarly, if $y > y_0$ then a necessary and sufficient condition for all veto players to weakly prefer the proposal to the status quo is that the *leftmost* veto player x_{Vl} weakly prefers it, i.e., $V_{Vl}(s, y) - V_{Vl}(0, y_0) \geq 0 \iff y \leq y_0 + \frac{s}{2|x_{Vl}|} = z_R(s)$. We thus have the following.

Definition 3 *A policy (s, y) with score s and ideology y (and hence quality $q = s + y^2$) is weakly preferred by all veto players to the statos quo i.f.f. $s \geq 0$ and $y \in Y_V(s) = [z_L(s), z_R(s)]$, where*

$$z_L(s) = y_0 - \frac{s}{2|x_{Vr}|} \quad and \quad z_R(s) = y_0 + \frac{s}{2|x_{Vl}|}.$$

We term $Y_V(s) = [z_L(s), z_R(s)]$ the *veto-proof interval given s* and $Y_V = \{(s, y) : s \geq 0, y \in Y_V(s)\}$ the *veto-proof set*.

In the policymaking stage, the decisionmaker is an agenda-setter vis-a-vis the veto players. As is customary in agenda-setting models, we henceforth restrict attention to strategy profiles in which both veto players break indifference in favor of the decisionmaker's proposal. With this restriction, the organization must always choose a policy (s, y) that maximizes the score (i.e. decisionmaker's utility) from within the subset of feasible policies $\mathbf{b} \cup b_0$ in the veto-proof set Y_V. A policy outside the veto-proof set can never prevail because it will be vetoed by one of the veto players. The decisionmaker will never propose a policy from within the set that doesn't maximize his utility, because any other proposal within the set will be accepted for sure. A formal statement of policy outcomes given each history is as follows.

Observation 1 *When the veto players break indifference in favor of the decisionmaker's proposal, a probability distribution over outcomes $w(\mathbf{b})$ can result from an equilibrium of the subgame commencing with $\mathbf{b}$ if and only if $\forall b$ in the support of $w(\mathbf{b})$, $(s,y) \in \underset{\{\mathbf{b} \cup b_0\} \cap Y_V}{\arg\max} s$.*

B.1   Necessary and Sufficient Equilibrium Conditions

An entrepreneur's pure strategy $b_i = (s_i, y_i)$ is a two-dimensional element of the set

$$\mathbb{B} \equiv \left\{ (s,y) \in R^2 \mid \left(s - y_0^2\right) + y^2 \geq 0 \right\},$$

or the set of scores and ideologies that imply positive-quality policies. A mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$.

————-

[[WANT TO DO RESTRICTION TO VETO PROOF SET HERE XX. JUST ASSUME I HAVE HERE AND DEAL WITH LATER. XX. A SLIGHT HEADACHE BUT MAKES EVERYTHING EASIER LATER]]

————

Having restricted the strategy space to the set of veto-proof policies Y_V, let $F_i(s)$ denote the CDF over scores induced by i's mixed strategy σ_i; we henceforth further restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.

We now derive necessary and sufficient equilibrium conditions in a series of lemmas. Note that $x_{Vl} < 0 < x_{Vr}$ implies that the status quo $(0, y_0)$ is both the unique veto-proof policy with 0-score and with 0-quality. Now let $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ denote i's expected utility for developing a policy $(s_i, y_i) \in Y_V$ if a tie would be broken in her favor. Clearly this is i's expected utility from developing any policy with $s_i \geq 0$ where $-i$ has no atom, and i can always achieve utility arbitrarily close to $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ by developing an $\varepsilon-$higher score policy. Now $\bar{\Pi}_i(s_i, y_i; \sigma_{-i}) =$

$$- \alpha_i \left(s - y_0^2 + y^2 \right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int\limits_{s_{-i} > s_i} V_i(s_{-i}, y_{-i}) \, d\sigma_{-i}. \tag{2}$$

The first term is the up-front cost of generating the quality. With probability $F_{-i}(s_i)$, i's opponent develops a policy with a lower score, i's policy in this case will then be proposed and passed for sure, and this yields utility $V_i(s_i, y_i)$. With the remaining probability $-i$'s policy will be proposed and passed for sure, yielding utility $V_i(s_{-i}, y_{-i})$.

Note that only the first two terms of equation 2 are affected by y_i. Taking the first derivative w.r.t. y_i yields $-2\alpha_i y_i + 2F_{-i}(s_i) x_i$, which is strictly decreasing in y_i. Given s_i, there is thus a unique strictly optimal value of y_i in the veto-proof interval $[z_L(s_i), z_R(s_i)]$, yielding our first key lemma.

4

Lemma 2 *At any score $s \geq 0$ where $F_{-i}(\cdot)$ has no atom, the policy $(s, y_i^*(s))$ is the strictly best score-s veto-proof policy, where*

$$y_i^*(s) = \frac{x_i}{\alpha_i} \cdot \min \left\{ \max \left\{ \frac{z_{-i}(s)}{x_i/\alpha_i}, F_{-i}(s) \right\}, \frac{z_i(s)}{x_i/\alpha_i} \right\}.$$

Lemma 2 states that for almost every score $s > 0$, proposer i's best combination of ideology and quality to generate a veto-proof policy with that score is unique. The expression in the lemma is equivalent to

$$y_i^*(s) = \begin{cases} z_L(s) & \text{if } \frac{x_i}{\alpha_i} F_{-i}(s) < z_L(s) \\ \frac{x_i}{\alpha_i} F_{-i}(s) & \text{if } z_L(s) \leq \frac{x_i}{\alpha_i} F_{-i}(s) \leq z_R(s) \\ z_R(s) & \text{if } z_R(s) < \frac{x_i}{\alpha_i} F_{-i}(s) \end{cases}.$$

The expression in the lemma will be more convenient to use in subsequent analysis.

The second Lemma establishes that in equilibrium there is 0 probability of a tie at strictly positive scores. The absence of score ties is an intuitive consequence opposing ideological interests and the fact that generating quality is all pay – at least one proposer will find it in her interests to either invest up-front in a bit more quality and make an ideological proposal weakly better than the expected outcome from a tie, or drop out of the contest.

Lemma 3 *In equilibrium there is 0-probability of a tie at scores $s > 0$.*

Proof (Sketch): Suppose not. Let p_i^s denote the atoms, and $\bar{y}^s$ denote the expected ideological outcome conditional on a tie. Also let $y_D(s) = \max \{\min \{0, z_R(s)\}, z_L(s)\} = \begin{cases} z_L(s) & \text{if } z_L(s) > 0 \\ 0 & \text{if } z_L(s) \leq 0 \leq z_R(s) \\ z_R(s) & \text{if } z_R(s) < 0 \end{cases}$,

which is the best veto-proof score-s ideology for the decisionmaker, and is therefore cheapest for the entrepeneurs to develop. (Absent VPs this would be $(s, 0)$). Also observe that $z_i(s)$ is the boundary of the veto-proof set on the same side of the status quo as entrepreneur i is from the DM. Now each entrepreneur can achieve her eqilibrium utility by mixing according to her strategy conditional on developing a score-s policy.

Suppose first $y_D(s) \neq \bar{y}^s$; then $V_k(s, y_D(s)) > V_k(s, \bar{y}^s)$ for some k. Then k is strictly better off deviating to developing $(s, y_D(s))$ with probablity $\frac{p_{-k}}{F_{-k}(s)}$ and $(s, E[y_k|s])$ with probability $1 - \frac{p_{-k}}{F_{-k}(s)}$ (and always winning at the tie).

Suppose next $y_D(s) = \bar{y}^s$. If some k develops a score s policy other than $y_D(s)$ with strictly positive probability, then she is also strictly better off deviating to developing $(s, y_D(s))$ with probability $\frac{p_{-k}}{F_{-k}(s)}$ and $(s, E[y_k|s])$ with probability $1 - \frac{p_{-k}}{F_{-k}(s)}$ (and always winning at the tie).

Finally suppose both players develop $y_D(s)$ at the atom. If $y_D(s)$ is on the boundary of the veto-proof set then for some k it is the worst veto-proof ideology among *all* veto-proof policies with

scores $\in [0, s]$. So this k is strictly better off deviating to $(0, y_0)$. If $y_D(s)$ is not on the boundary then $y_D(s) = 0$ and there is a strictly best score$-s$ policy for some player to deviate to and win for sure. **QED**

Lemmas 2 − 3 jointly imply that in equilibrium, proposer i can compute her expected utility as if her opponent only makes proposals of the form $(s_{-i}, y^*_{-i}(s_{-i}))$. The utility from making *any* proposal (s_i, y_i) with $s_i \geq 0$ where $-i$ has no atom (or a tie would be broken in i's favor) is therefore $\bar{\Pi}^*_i(s_i, y_i; F) =$

$$- \alpha_i \left(s_i - y_0^2 + y_i^2 \right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i \left(s_{-i}, y^*_{-i}(s_{-i}) \right) dF_{-i}, \tag{3}$$

and her utility from making the *best* proposal with score s_i (where $-i$ has no atom or a tie would be broken in her favor) is $\bar{\Pi}^*_i(s_i, y^*_i(s_i); F)$, which we henceforth denote $\bar{\Pi}^*_i(s_i; F)$, which is right-continuous because CDFs are right continuous.

Next we show the following *score optimality* property, which simplifies the characterization of equilibria.

Lemma 4 *For all i and $\hat{s}$ in the support of F_i, $\bar{\Pi}^*_i(\hat{s}; F) = \max_s \left\{ \bar{\Pi}^*_i(s; F) \right\}$.*

Proof (Sketch): We first argue that $\lim_{s \to \hat{s}^-} \left\{ \bar{\Pi}^*_i(s; F) \right\} \leq \bar{\Pi}^*_i(\hat{s}; F)$ for $\hat{s} > 0$; that is, it is weakly better to just win than to just lose at a strictly positive score. If $-i$ has no atom at $\hat{s}$ then it is equivalent. Suppose $-i$ has an atom at $\hat{s}$; then (by no ties) i does not, and at the atom $-i$ develops $(\hat{s}, y^*_{-i}(\hat{s}))$. Let $y_i^{\hat{s}-} = \lim_{s \to \hat{s}^-} \left\{ y^*_i(s) \right\}$ denote i's optimal ideology if she were to just lose at score $\hat{s}$. Then $\bar{\Pi}^*_i \left(\hat{s}, y_i^{\hat{s}-}; F \right) - \lim_{s \to \hat{s}^-} \left\{ \bar{\Pi}^*_i(s; F) \right\} = p^{\hat{s}}_{-i} \left(V_i \left(\hat{s}, y_i^{\hat{s}-} \right) - V_i \left(\hat{s}, y^*_{-i}(\hat{s}) \right) \right) \geq 0$ since $V_i \left(\hat{s}, y_i^{\hat{s}-} \right) \geq V_i(\hat{s}, y_D(s)) \geq V_i \left(\hat{s}, y^*_{-i}(\hat{s}) \right)$, recalling that $y_D(\hat{s})$ is the best veto-proof score-$\hat{s}$ policy for the decisionmaker. The first weak inequality comes from the fact that $-i$ has no atom in an interval below $\hat{s}$ and lemma 2 applied to i; the second weak inequality comes from the fact that i has no atom at $\hat{s}$ and lemma 2 applied to $-i$. Finally $\bar{\Pi}^*_i(\hat{s}; F) \geq \bar{\Pi}^*_i \left(\hat{s}, y_i^{\hat{s}-}; F \right)$.

We now prove the main statement. Let U^*_i denote i's utility form playing according to her strategy. Since i can achieve utility arbitrarily close to $\bar{\Pi}^*_i(s; F) \; \forall s$, we have $\bar{\Pi}^*_i(s; F) \leq U^*_i \; \forall s$ since otherwise i would have a profitable deviation. We now argue $U^*_i \leq \bar{\Pi}^*_i(\hat{s}; F)$ for $\hat{s}$ in i's support and hence $U^*_i = \bar{\Pi}^*_i(\hat{s}; F)$. Consider any such $\hat{s}$, and observe that $\Pi^*_i(s; F) = \bar{\Pi}^*_i(s; F)$ in a neigborhood above and below $\hat{s}$. [Note: we'll need to define $\Pi^*_i(s; F)$ somewhere before this].

- Suppose $-i$ does not have an atom at $\hat{s}$. Then $\Pi^*_i(\hat{s}; F) = \bar{\Pi}^*_i(\hat{s}; F)$ and is continuous in a neighborhood around $\hat{s}$ so $U^*_i \leq \bar{\Pi}^*_i(\hat{s}; F)$ since otherwise i would have a profitable deviation.

- Suppose $-i$ has an atom at $\hat{s}$; then i does not. If i has support in a neighborhood above $\hat{s}$ then $U_i^* \le \bar{\Pi}_i^* (s; F)$ (by right continuity) since otherwise i would have a profitable deviation. If i has support in a neighborhood below $\hat{s}$ then $U_i^* \le \lim_{s \to \hat{s}^-} \{\bar{\Pi}_i^* (s; F)\}$ since otherwise i would have a profitable deviation; but then $\lim_{s \to \hat{s}^-} \{\bar{\Pi}_i^* (s; F)\} \le \bar{\Pi}_i^* (\hat{s}; F) \to U_i^* \le \bar{\Pi}_i^* (\hat{s}; F)$.

Lastly since $\bar{\Pi}_i^* (s; F) \le U_i^*$ $\forall s$ and $\bar{\Pi}_i^* (\hat{s}; F) = U_i^*$ at all $\hat{s}$ in i's support we must have $\bar{\Pi}_i^* (\hat{s}; F) = \max_s \{\bar{\Pi}_i^* (s; F)\}$. QED

B.2   Form of Equilibrium Proof Sketches

B.2.1   Support bounded above

Do I need this? Probably. Use old proof. [[FILL ME IN]]

B.2.2   Pull policy closer

If $|x_i - y_i^* (s)| > |x_i - y_0|$ then $\bar{\Pi}_i^* (s; F) - \bar{\Pi}_i^* (0; F) < 0$.

By contrapositive this implies $\hat{s} \in \text{supp}^+ \{F_i\} \to |x_i - y_i^* (s)| \le |x_i - y_0|$. In other words, at any score in the support, the optimal policy must be weakly closer to the entrepreneur than the status quo. For example, this rules out $y_i^* (s) = z_{-i} (s)$ at a positive support point, where $z_{-i} (s)$ is the "wrong" boundary of the veto-proof set for player i. (This is effectively a restriction on the score CDF $F_{-i} (s)$ of the disadvantaged entrepeneur that it cannot be so low that the advantaged entrepreneur would optimally work at or further away than the status quo. It also implies restrictions on when there can be a mixed-strategy equilibrium.)

Proof:

Suppose $|x_i - y_i^* (s)| > |x_i - y_0|$; from the definition of $y_i^* (s)$ it must be that $sign(x_i) = sign(y_0)$ (i is on the same side of the DM as the status quo) and $F_i (s) < \frac{y_0}{x_i/\alpha_i}$ (should this be weak?). Now

$$
\begin{aligned}
\bar{\Pi}_i^* (s, y; F) - \bar{\Pi}_i^* (0; F) &= -\alpha_i \left(s - y_0^2 + y^2\right) + \int_0^s \left(V_i (s, y) - V_i \left(s_{-i}, y_{-i}^* (s_{-i})\right)\right) dF_{-i} \\
&\le -\alpha_i \left(s - y_0^2 + y^2\right) + \int_0^s \left(V_i (s, y) - V_i (s_{-i}, z_{-i} (s_{-i}))\right) dF_{-i} \\
&\le -\alpha_i \left(s - y_0^2 + y^2\right) + \int_0^s \left(V_i (s, y) - V_i (s, z_{-i} (s))\right) dF_{-i} \text{ (since } \frac{\partial V_i (s, z_{-i} (s))}{\partial s} < 0) \\
&= -\alpha_i s + \alpha_i \left(-\left(y^2 + y_0^2\right) + F_{-i} (s) \frac{x_i}{\alpha_i} \cdot 2 \left(y - z_{-i} (s)\right)\right)
\end{aligned}
$$

Beyond this the sketch is simple; the above at the maximizer $y_i^*(s)$ is strictly increasing in $F_i(s)$ (through an envelope argument) unless you are on the wrong boundary (then it is obviously strictly negative anyway). Then you show at the $F_i(s)$ s.t. $y_i^*(s) = y_0$ the expression is weakly negative. Below are the annoying details.

Let

$$G_i(s, y; F_{-i}) = -\alpha_i s + \alpha_i \left(-\left(y^2 + y_0^2\right) + F_{-i} \frac{x_i}{\alpha_i} \cdot 2\left(y - z_{-i}(s)\right) \right).$$

So we have

- $y_i^*(s) = \underset{y \in [z_L(s), z_R(s)]}{\arg \max} \{G_i(s, y; F_{-i}(s))\}$ and $G_i(s, y_i^*(s); F_{-i}(s)) = \bar{\Pi}_i^*(s; F) - \bar{\Pi}_i^*(0; F)$

- $F_{-i} \frac{x_i}{\alpha_i} = \arg \max \{G_i(s, y; F_{-i})\}$

- $G_i(s, y; F_{-i})$ is strictly increasing in F_{-i} $\forall y \in [z_L(s), z_R(s)]$ that is $\neq z_{-i}(s)$

Now if $F_{-i}(s) \leq \frac{z_{-i}(s)}{x_i/\alpha_i}$ then $y_i^*(s) = z_{-i}(s)$ and $G_i(s, y_i^*(s); F_{-i}(s)) = -\alpha_i \left(s - y_0^2 + [z_{-i}(s)]^2\right) < 0$. If $F_i(s) \in \left(\frac{z_{-i}(s)}{x_i/\alpha_i}, \frac{y_0}{x_i/\alpha_i} \right)$ so $y_i^*(s) \neq z_{-i}(s)$ then

$$G_i(s, y_i^*(s); F_{-i}(s)) < G_i\left(s, y_i^*(s); \frac{y_0}{x_i/\alpha_i}\right) \leq G_i\left(s, y_0; \frac{y_0}{x_i/\alpha_i}\right) = -\alpha_i s \cdot \left(1 - \frac{y_0}{x_{V_i}}\right) \leq 0$$

(since $y_0 \in [x_{Vl}, x_{Vr}]$ and $sign(y_0) = sign(x_i) = sign(x_{Vi})$). QED

(Note: the above arguments also imply that playing exactly at the status quo cannot be in the support either unless $y_0 = x_{V_i}$. I believe its also true that if $y_0 = x_{V_i}$ then the sometimes inactive entrepreneur *must* enter at exactly the status quo in all MS equilibria).

B.2.3 Notation

- Let $\underline{s}_i = \min \{\mathrm{supp}\{F_i\}\}$. Let $k \in \arg \max \{\underline{s}_i\}$ (so k is the player with the highest minimum support point in a strategy profile).

- Let $\underset{S}{supp}\{F_i\} = supp\{F_i\} \cap S$ denote i's support points in S.

- Let

$$Z_i(s; F) = -\alpha_i \left(s - y_0^2 + (z_i(s))^2\right) + F \cdot V_i(s, z_i(s)),$$

so

$$\begin{aligned} Z_i'(s; F) &= -\alpha_i \left(1 + \frac{z_i(s)}{-x_{V-i}}\right) + F \cdot \left(1 + \frac{x_i}{-x_{V-i}}\right) \\ &= -(\alpha_i - F) + \frac{\alpha_i}{-x_{V-i}} \left(F \frac{x_i}{\alpha_i} - z_i(s)\right) \end{aligned}$$

It is easily verified that $Z_i''(s; F) < 0$ so $Z_i(s; F)$ is strictly concave in s.

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B.2.4 Corrected (Original) Augmented Gap Lemma

(a) If $\hat{s} \in \text{supp}^+ \{F_i\}$ then $F_{-i}(\hat{s}) > 0$.

(b) If $\hat{s} \in \text{supp}^+ \{F_i\}$ and $\exists \tilde{s} < \hat{s}$ s.t. $F_{-i}(\tilde{s}) = F_{-i}(\hat{s})$, then $y_i^*(\hat{s}) = z_i(\hat{s})$ and $\underset{\{s:F_{-i}(s)=F_{-i}(\hat{s})\}}{supp} \{F_i\} = \hat{s}$. (In words, if i has support at $\hat{s}$ and $-i$ there has a flat spot, then this is i's only support over the entire flat spot, and the optimal ideology at $\hat{s}$ is on the boundary).

(Note: Immediately implies $y_i^*(s) = z_i(s) \ \forall s$ s.t. $F_{-i}(s) = F_{-i}(\hat{s})$).
(Note: Immediately implies for $\tilde{s} < \hat{s}$ both in $\text{supp}\{F_i\}$, $F_{-i}(\tilde{s}) < F_i(\hat{s})$).

(c) If $\hat{s} \in \text{supp}^+ \{F_i\}$ and $\exists \tilde{s} < \hat{s}$ s.t. $F_{-i}(\tilde{s}) = F_{-i}(\hat{s})$, then $\hat{s} = \underline{s}_i$.
(Note: Immediately implies that $\hat{s} > \underline{s}_i$ and $\hat{s} \in \text{supp}\{F_i\} \to F_{-i}(s) < F_{-i}(\hat{s}) \ \forall s < \hat{s}$.

Proof of (a): Very simple; fill later.

Proof of (b): Suppose $\hat{s} \in \text{supp}^+ \{F_i\}$ and $\exists \tilde{s} < \hat{s}$ s.t. $F_{-i}(\tilde{s}) = F_{-i}(\hat{s})$. Can't have $y_i^*(\hat{s}) = z_{-i}(s)$ by previous lemma, and if $y_i^*(\hat{s}) \neq z_i(s)$ then interior and argument for $-i$ having support immediately below is same as w-out VPs. So $y_i^*(\hat{s}) = z_i(\hat{s})$ and also $y_i^*(s) = z_i(s) \ \forall s \in [\tilde{s}, \hat{s}]$ (from the definition of y_i^*).

Now for all $s \in [\tilde{s}, \hat{s}]$,

$$\bar{\Pi}_i^*(s; F) = Z_i(s; F_{-i}(\hat{s})) + \int_{\hat{s}}^{\infty} V_{-i}\left(s_{-i}, y_{-i}^*(s_{-i})\right) f(s_{-i}) ds_{-i}$$

so $\frac{\partial \bar{\Pi}_i^*(s;F)}{\partial s} = Z_i'(s; F_{-i}(\hat{s}))$ and over $[\tilde{s}, \hat{s}]$ we have $\bar{\Pi}_i^*(s; F)$ strictly concave and has unique maximum at $\hat{s}$ where $Z_i'(\hat{s}; F_{-i}(\hat{s})) \geq 0$, and no other points in $[\tilde{s}, \hat{s}]$ can be in the support.

Proof of (c): By contradiction. Suppose $\hat{s} \in \text{supp}^+ \{F_i\}$, there $\exists \tilde{s} < \hat{s}$ s.t. $F_{-i}(\tilde{s}) = F_{-i}(\hat{s})$, but $\underline{s}_i < \hat{s}$. Then by **(b)**, $y_i^*(\hat{s}) = z_i(\hat{s})$ and there $\exists s_i' = \max\left\{\underset{s<\hat{s}}{supp}\{F_i\}\right\}$ where $s_i' < \hat{s}$ and $F_{-i}(s_i') < F_{-i}(\hat{s})$. Now since i has no support in $(s_i', \hat{s})$, also by **(b)** $y_{-i}^*(s_{-i}) = z_{-i}(s_{-i}) \ \forall s_{-i} \in [s_i', \hat{s})$ in $-i$'s support, and by the premise $-i$ has no atom at $\hat{s}$. Thus $\bar{\Pi}_i^*(s_i'; F)$ may be written as

$$\bar{\Pi}_i^*(s_i'; F) = Z_i(s_i'; F_{-i}(\hat{s})) + \int_{s_i'}^{\hat{s}} \left(V_i\left(s_{-i}, z_{-i}^*(s_{-i})\right) - V_i\left(s_i', y_i^*(s_i')\right)\right) dF_{-i} + \int_{\hat{s}}^{\infty} V_{-i}\left(s_{-i}, y_{-i}^*(s_{-i})\right) f(s_{-i}) ds_{-i}$$

and

$$\bar{\Pi}_i^*(\hat{s}; F) - \bar{\Pi}_i^*(s_i'; F) = \int_{s_i'}^{\hat{s}} Z_i'(s; F_{-i}(\hat{s})) ds + \int_{s_i'}^{\hat{s}} \left(V_i\left(s_i', y_i^*(s_i')\right) - V_i\left(s_{-i}, z_{-i}^*(s_{-i})\right)\right) dF_{-i}$$

Now the first term is strictly positive since from the preceding step $Z_i''(s; F_{-i}(\hat{s})) < 0$ and $Z_i'(\hat{s}; F_{-i}(\hat{s})) \geq 0$. The entire expression is thus strictly positive as long as the second term is weakly positive. For this

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it suffices to show $V_i\left(s_i', y_i^*\left(s_i'\right)\right) - V_i\left(s_i', z_{-i}^*\left(s_i'\right)\right) \geq 0$ (which is immediate) and $\frac{\partial}{\partial s}\left(V_i\left(s, z_{-i}^*\left(s\right)\right)\right) = 1 - \frac{x_i}{x_{Vi}} \leq 0$ (that is, i is getting weakly worse off as $-i$ moves up her respective boundary of the veto-proof set). This in turn is the case i.f.f. $x_i \geq x_{Vi}$; that is, entrepreneur i is weakly more extreme than the same-sided veto player, as assumed. **QED.**

B.2.5 Atoms Preliminary Observations

Suppose the highest minimum support point $\underline{s}_k$ is > 0. Then in any equilibrium (pure or mixed) the properties of $-k$ are: $\underline{s}_{-k} = 0$, $-k$ has an atom at 0, no support in $(0, \underline{s}_k)$, and no atom at $\underline{s}_k$.

   Proof:

   First $\underline{s}_k > 0 \rightarrow F_{-k}\left(\underline{s}_k\right) > 0$ by gap lemma.

   Now can't have $\underline{s}_{-k} = \underline{s}_k$ since no gap implies both have atoms there, which is not possible by no ties. Also can't have $s \in (0, \underline{s}_k) \in \text{supp}\{F_{-i}\}$ by no gap since at all such points $F_k\left(\cdot\right) = 0$. So $\underline{s}_{-k} = 0$ and $-k$ must there have an atom.

   It must also be that $-k$ doesn't have an atom at $\underline{s}_k$; if she did then $\underline{s}_k$ would be in her support but not the lowest point, so by no gap k would need an atom there too (since $\underline{s}_k$ is the lowest point in k's support), contradicting no ties.

B.2.6 More Notation (Monopoly Results):

Let

$$\hat{y}_i^* = \frac{1}{\alpha_i} x_i + \left(1 - \frac{1}{\alpha_i}\right) x_{V_{-i}}$$

denote the weighted midpoint between entrepreneur i and her binding veto player $x_{V_{-i}}$, and let

$$\hat{s}_i^* = 2\left(-x_{V_{-i}}\right) \cdot \left(\hat{y}_i^* - y_0\right)$$

Observe that $\hat{s}_i^*$ is the unique value s.t. $Z'\left(\hat{s}_i^*; 1\right) = 0 \iff z_i\left(\hat{s}_i^*\right) = \hat{y}_i^*$. The monopoly score is then

$$s_i^* = \max\left\{\hat{s}_i^*, 0\right\}$$

It is also helpful to characterize $\bar{s}_i > \hat{s}_i^*$ s.t. $z_i\left(\bar{s}_i\right) = \frac{x_i}{\alpha_i}$; this is the score s.t. the ideology on own boundary of the veto-proof set is equal to the weighted midpoint with the DM. If an entrepreneur will win for sure, the optimal ideology at scores above this is no longer on the boundary. We have

$$\bar{s}_i = 2\left(-x_{V-i}\right)\left(\frac{x_i}{\alpha_i} - y_0\right)$$

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B.3 PS Equilibria

Suppose $\underset{s > \underline{s}_k}{supp}\{F_i\} = \emptyset \ \forall i$, so at least k is playing a PS at $\underline{s}_k$.

B.3.1 Suppose $\underline{s}_k = 0$

Then $\underline{s}_{-k} = 0$, and both must be playing a pure strategy at 0. The condition for this to be an equilibrium is that neither wants to generate a positive-score policy as a monopolist, $\max_i\{\hat{s}_i^*\} \leq 0$ $\forall i \iff Z_i'(0;1) \leq 0 \ \forall i$. In the primitive parameters this reduces to:

$$\frac{1}{\alpha_R}x_R + \left(1 - \frac{1}{\alpha_R}\right)x_{VL} \leq y_0 \leq \frac{1}{\alpha_L}x_L + \left(1 - \frac{1}{\alpha_L}\right)x_{VR}$$

B.3.2 Suppose $\underline{s}_k > 0$.

Then by (**atoms prelim**) $\underline{s}_{-k} = 0$, $-k$ has an atom at 0, no support in $(0, \underline{s}_k)$, and no atom at $\underline{s}_k$. Finally since $\underset{s > \underline{s}_k}{supp}\{F_i\} = \emptyset \ \forall i$, $-k$ must also be playing a pure strategy at $\underline{s}_{-k} = 0$. Conditions for this to be an equilibrium are as follows.

First, some player must want to play a strictly positive monopoly score, i.e. $\max_i\{\hat{s}_i^*\} > 0 \iff \max_i\{Z_i'(0;1)\} > 0 \ \forall i$. Next, k can only be the player with the highest monopoly score ($k = \arg\max_i\{\hat{s}_i^*\}$), and her monopoly score must be *strictly* higher than her opponent's monopoly score, i.e.

$$\underline{s}_k = \hat{s}_k^* > \hat{s}_{-k}^*$$

[[xx This will be apparent when we write the conditions xx]].

Finally, $-k$ must not want to enter at exactly $\hat{s}_k^*$ and just beat k. Noting that $y_{-k}^*(\hat{s}_k^*) = \frac{x_{-k}}{\alpha_{-k}} \cdot \min\left\{\max\left\{\frac{z_k(s)}{x_{-k}/\alpha_{-k}}, 1\right\}, \frac{z_{-k}(s)}{x_{-k}/\alpha_{-k}}\right\}$, equilibrium requires that

$$\bar{\Pi}_{-k}^*(\hat{s}_k^*; F) - \bar{\Pi}_{-k}^*(0; F) \leq 0$$

In the analysis of mixed strategy equilibria under (**Sometimes Inactive Atom**) we derive a function $\bar{y}_{-k}(\underline{s})$; the condition for pure strategy equilibrium employs this function and is just

$$\frac{\bar{y}_{-k}(\hat{s}_k^*)}{x_{-k}/\alpha_{-k}} \geq 1$$

Intuitively, $\bar{y}_{-k}(\hat{s}_k^*)$ is what $-k$'s unbounded optimum would have to be to make her *indifferent* between staying out and entering optimally at $\hat{s}_k^*$, and so $\frac{\bar{y}_{-k}(\hat{s}_k^*)}{x_{-k}/\alpha_{-k}}$ is the value of $F_k(\hat{s}_k^*)$ that would generate this indifference. If this is < 1, then $-k$ would surely enter if she would win for sure by doing so, and if this is ≥ 1 then even winning for sure would be insufficient to induce entry.

B.4 MS Equilibria

Suppose $\underset{s>\underline{s}_k}{supp}\{F_i\} \neq \emptyset$ for some i.

B.4.1 MS Equilibria, Basic Form

We have $\underset{s\geq\underline{s}_k}{supp}\{F_i\} = \underset{s\geq\underline{s}_k}{supp}\{F_{-i}\}$ and is convex, positive measure, and includes $\underline{s}_k$.

Proof (virtually identical to AER):

(Common Support strictly above $\underline{s}_k$) $\underset{s>\underline{s}_k}{supp}\{F_i\} = \underset{s>\underline{s}_k}{supp}\{F_{-i}\}$. Trivial; by no gap any $s_i > \underline{s}_k \geq \underline{s}_i$ is $\in \text{supp}\{F_{-i}\}$ and true both ways.

(Support strictly above $\underline{s}_k$ is convex, has no space between it and $\underline{s}_k$)

Suppose the common support above $\underline{s}_k$ were not convex, or that it had a gap with $\underline{s}_k$. Then there would exist an $s' > \underline{s}_k$ not in the common support, and an $s'' = \underset{s>s'}{\min}\{\text{supp}\{F_k\}\}$ in the common support s.t. $F_i(s) = F_i(s') \ \forall i$ and $\forall s \in [s', s'')$. Since by no ties both cannot have atoms at s'' there is some j s.t. $F(s'') = F_j(s')$, violating no gap.

B.4.2 MS Equilibria, Show Atomless & Differential Equations

Suppose $\underset{s>\underline{s}_k}{supp}\{F_i\} \neq \emptyset$ for some i, so $\underset{s\geq\underline{s}_k}{supp}\{F_i\} = \underset{s\geq\underline{s}_k}{supp}\{F_{-i}\} = [\underline{s}, \bar{s}]$. For any $s \in (\underline{s}, \bar{s}]$ we have $y_i^*(s) \neq z_{-i}(s)$, implying $V_i(s, y_i^*(s)) > V_i(s, y_{-i}^*(s))$. This then implies that the CDFs must be continuous over $[\underline{s}, \bar{s}]$ and therefore there are no atoms in $(\underline{s}, \bar{s}]$, since an atom would generate a discontinuity and violate $\bar{\Pi}_i^*(s; F)$ constant over $[\underline{s}, \bar{s}]$. Thus within the mixing interval we have $\frac{\partial}{\partial s}\bar{\Pi}_i^*(s; F) = 0$ so...

$$\max\left\{-(\alpha_i - F_{-i}(s)), Z_i'(s; F_{-i}(s))\right\} + f_{-i}(s)\left(V_i(s, y_i^*(s)) - V_i(s, y_{-i}^*(s))\right) = 0$$

and also $F_{-i}(\bar{s}) = F_i(\bar{s}) = 1$.

B.5 MS Equilibria, Boundary Conditions at Bottom

Like PS, there are two cases.

B.5.1 Suppose $\underline{s}_k > 0$

By (**Atoms Preliminary Observations**) $\underline{s}_{-k} = 0$, $-k$ has an atom at 0, no support in $(0, \underline{s}_k)$, and no atom at $\underline{s}_k$. We now argue k must have an atom at $\underline{s}_k$. From preceding step, $\underline{s}_k \in \text{supp}\{F_{-k}\}$; so

by no gap $F_k(\underline{s}_k) > 0$, yielding the atom. Summarizing, k has support over $[\underline{s}, \overline{s}]$ and an atom at $\underline{s}$, while $-k$ has support over $\{0\} \cup [\underline{s}, \overline{s}]$ and an atom at 0.

We now derive the conditions that must hold at the two atoms.

(Always active atom) We first derive $F_{-k}(0) = F_{-k}(\underline{s})$. At $\underline{s}$ we must have $y_k^*(\underline{s}) = z_k(\underline{s})$ and therefore $y_k^*(s) = z_k(s) \; \forall s \in [0, \underline{s}]$. So for $s \in [0, \underline{s}]$ we have $\frac{\partial \bar{\Pi}_k^*(s; F)}{\partial s} = Z_i'(s; F_{-k}(\underline{s}))$. Now $\underline{s} \in \mathrm{supp}\{F_k\} \rightarrow Z_i'(\underline{s}; F_{-k}(\underline{s})) \geq 0$ and also (fill in check details XX) from differential equation we require $Z_i'(\underline{s}; F_{-k}(\underline{s})) \leq 0$ so $Z_i'(\underline{s}; F_{-k}(\underline{s})) = 0$ which implies:

$$F_{-k}(\underline{s}) = \alpha_k \left(\frac{z_k(\underline{s}) - x_{V_{-k}}}{x_k - x_{V_{-k}}} \right)$$

Note of course that $F_{-k}(\hat{s}_k^*) = 1$ so we must have $\underline{s} \in (0, \hat{s}_k^*)$.

(Sometimes active atom) At $\underline{s}$ we must have that $\bar{\Pi}_{-k}^*(0; F) = \bar{\Pi}_{-k}^*(\underline{s}; F)$, or (the value of this rewriting will soon be apparent)

$$\frac{1}{\alpha_{-k}} \left(\bar{\Pi}_{-k}^*(0; F) - \bar{\Pi}_{-k}^*(\underline{s}, y_0; F) \right) = \frac{1}{\alpha_{-k}} \left(\bar{\Pi}_{-k}^*(\underline{s}; F) - \bar{\Pi}_{-k}^*(\underline{s}, y_0; F) \right)$$

The l.h.s. represents the net "cost" of beating k's policy $(\underline{s}, z_{-k}(\underline{s}))$ at $\underline{s}$ with an ideology exactly at the status quo $(\underline{s}, y_0)$. (This can actually be positive, in which case the equality can't be satisfied). Once this is done it makes more veto-proof policies become feasible, and the r.h.s. presents the net benefit of moving from $(\underline{s}, y_0)$ to the optimal one $(\underline{s}, y_{-k}^*(\underline{s}))$. (This is always at least weakly positive).

We derive the "cost" term first, which is:

$$
\begin{aligned}
\frac{1}{\alpha_{-k}} \left(\bar{\Pi}_{-k}^*(0; F) - \bar{\Pi}_{-k}^*(\underline{s}, y_0; F) \right) &= \frac{1}{\alpha_{-k}} \left(\alpha_{-k}\underline{s} - F_k(\underline{s}) \cdot (V_{-k}(\underline{s}, y_0) - V_{-k}(\underline{s}, z_k(\underline{s}))) \right) \\
&= \frac{1}{\alpha_{-k}} \left(\alpha_{-k}\underline{s} - F_k(\underline{s}) \, 2x_{-k} \left(\frac{\underline{s}}{2x_{V-k}} \right) \right) \\
&= \underline{s} \left(1 - \frac{F_k(\underline{s}) \frac{x_{-k}}{\alpha_{-k}}}{x_{V-k}} \right)
\end{aligned}
$$

It is now clear that for this cost to actually be a cost requires that the unbounded optimum $F_k(\underline{s}) \frac{x_{-k}}{\alpha_{-k}}$ implied by the atom $F_k(\underline{s})$ be weakly further from $-k$ than the same-sided veto player x_{V-k}.

We next derive the benefit term, which is:

$$
\begin{aligned}
& \frac{1}{\alpha_{-k}} \left(\bar{\Pi}_{-k}^*(\underline{s}; F) - \bar{\Pi}_{-k}^*(\underline{s}, y_0; F) \right) \\
&= \frac{1}{\alpha_{-k}} \left(-\alpha_{-k} \left([y_{-k}(\underline{s})]^2 - y_0^2 \right) + F_k(\underline{s}) \cdot 2x_{-k} \left(y_{-k}(\underline{s}) - y_0 \right) \right) \\
&= 2 \left(y_{-k}(\underline{s}) - y_0 \right) \left(F_k(\underline{s}) \frac{x_{-k}}{\alpha_{-k}} - \left(\frac{y_{-k}(\underline{s}) + y_0}{2} \right) \right)
\end{aligned}
$$

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Now let $\bar{y}_i(s) = \frac{x_i}{\alpha_i} F_{-i}(s)$ denote i's *unconstrained optimum* at score s, which implies both $F_{-i}(s)$ (since $F_{-i}(s) = \frac{\bar{y}_i(s)}{x_i/\alpha_i}$) and $y_i(s)$ (since $y_i(s) = \min\{\max\{z_L(s), \bar{y}_i(s)\}, z_R(s)\}$).

Now the condition may be rewritten as:

$$\underline{s}\left(1 - \frac{\bar{y}_{-k}(\underline{s})}{x_{V-k}}\right) = 2(y_{-k}(\underline{s}) - y_0)\left(\bar{y}_{-k}(\underline{s}) - \left(\frac{y_{-k}(\underline{s}) + y_0}{2}\right)\right)$$

Now at a solution $\bar{y}_{-k}(\underline{s})$, the actual optimum $y_{-k}(\underline{s})$ can take two possible values: (i) if $\bar{y}_{-k}(\underline{s})$ is interior to the veto proof set then $y_{-k}(\underline{s}) = \bar{y}_{-k}(\underline{s})$, or (ii) $\bar{y}_{-k}(\underline{s})$ is outside the veto-proof set then $y_{-k}(\underline{s}) = z_{-k}(s)$. (We have already shown no $\bar{y}_{-k}(\underline{s})$ implying $y_{-k}(\underline{s}) = z_k(s)$ can satisfy the equality). We consider each possibility in turn.

(**Boundary Solution**) If $\bar{y}_{-k}(\underline{s})$ is outside the veto-proof set, then $y_{-k}(\underline{s}) = z_{-k}(\underline{s})$ and it solves the equality

$$\underline{s}\left(1 - \frac{\bar{y}_{-k}(\underline{s})}{x_{V-k}}\right) = 2(z_{-k}(\underline{s}) - y_0)\left(\bar{y}_{-k}(\underline{s}) - \left(\frac{z_{-k}(\underline{s}) + y_0}{2}\right)\right)$$

and substituting yields

$$1 - \frac{\bar{y}_{-k}(\underline{s})}{x_{V-k}} = \frac{1}{-x_{V_k}}\left(\bar{y}_{-k}(\underline{s}) - \left(\frac{z_{-k}(s) + y_0}{2}\right)\right)$$

From the above condition it is self evident that a simple solution to the quality exists that is (i) strictly increasing in $\underline{s}$, (ii) strictly closer to $-k$ than y_0 if $-k$ and y_0 are on the same side of the DM, and (iii) strictly closer to $-k$ than both y_0 and the DM if $-k$ and y_0 are on opposite sides of the status quo. Solving the equality yields the linear function

$$
\begin{aligned}
\bar{y}_{-k}(\underline{s}) &= y_0 + (-x_{Vk})\left(\frac{x_{V_{-k}} - y_0}{x_{V_{-k}} - x_{Vk}}\right) + \left(\frac{-x_{V_{-k}}/x_{Vk}}{x_{V_{-k}} - x_{Vk}}\right)\left(\frac{\underline{s}}{4}\right) \\
&= y_0 + (-x_{Vk})\left(\frac{x_{V_{-k}} - y_0}{x_{V_{-k}} - x_{Vk}}\right) + \frac{z'_{-k}(\underline{s})}{2\left(1 - \frac{x_{Vk}}{x_{V-k}}\right)} \cdot \underline{s}
\end{aligned}
$$

From the above, it is also clear that $\bar{y}_{-k}(\underline{s})$ is shallower than $z_{-k}(\underline{s})$, and therefore crosses it exactly once at some $\bar{s}_{-k} > 0$.

(**Interior Solution**) If $\bar{y}_{-k}(\underline{s})$ is inside the veto-proof set, then $y_{-k}(\underline{s}) = \bar{y}_{-k}(\underline{s})$ and it solves the equality

$$\underline{s}\left(1 - \frac{\bar{y}_{-k}(\underline{s})}{x_{V-k}}\right) = (\bar{y}_{-k}(\underline{s}) - y_0)^2$$

Clearly this also has a unique solution $\bar{y}_{-k}(\underline{s})$ that is strictly increasing in $\underline{s}$ with $\bar{y}_{-k}(0) = y_0$ and $\lim_{\underline{s} \to \infty} \hat{y}_{-k}(s) = x_{V-k}$. Moreover using implicit differentiation the derivative is

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$$\bar{y}'_{-k}(\underline{s}) = \frac{1 - \frac{\bar{y}_{-k}(\underline{s})}{x_{V-k}}}{2(\bar{y}_{-k}(\underline{s}) - z_k(s))}$$

so $\bar{y}_{-k}(\underline{s})$ satisfies Inada conditions: $\bar{y}'_{-k}(0) = \infty$, $\lim_{\underline{s}\to\infty} \bar{y}'_{-k}(\underline{s}) = 0$, and $\bar{y}''_{-k}(\underline{s}) < 0$.

(Combining Solutions) From the section on (Bounary solution) we have that the final $\bar{y}_{-k}(\underline{s})$ is equal to

$$\bar{y}_{-k}(\underline{s}) = y_0 + (-x_{Vk})\left(\frac{x_{V_{-k}} - y_0}{x_{V_{-k}} - x_{Vk}}\right) + \left(\frac{-x_{V_{-k}}/x_{Vk}}{x_{V_{-k}} - x_{Vk}}\right)\left(\frac{\underline{s}}{4}\right)$$

for $\underline{s} \in [0, \bar{s}_{-k}]$, where $\bar{s}_{-k} > 0$ denotes the strictly positive score at which $\bar{y}_{-k}(\underline{s})$ is equal to $z_{-k}(\underline{s})$ when the true optimum $y_{-k}(\underline{s})$ is conjectued to be on the boundary.

Conversely, for $\underline{s} > \bar{s}_{-k}$ the unbounded optimum that would make $-k$ indifferent if the true optimum were on the boundary is strictly further away from the entrepreneur than the boundary; this means at this unbounded optimum she would strictly gain by moving inward, and the solution must be interior and thus

$$\underline{s}\left(1 - \frac{\bar{y}_{-k}(\underline{s})}{x_{V-k}}\right) = (\bar{y}_{-k}(\underline{s}) - y_0)^2$$

Summarizing, it can be asserted that the final $\bar{y}_{-k}(\underline{s})$ satisfies

- $\bar{y}_{-k}(\underline{s}) \in (\max\{y_0, 0\}, x_{V-k})$

- $\bar{y}'_{-k}(\underline{s}) > 0$ and finite

- $\bar{y}_{-k}(\underline{s})$ is linear for $\underline{s} \in [0, \bar{s}_{-k}]$ and satisfies Inada conditions for $\underline{s} > \bar{s}_{-k}$, approaching x_{V-k} in the limit

(Solving for the Cutoff $\bar{s}_{-k}$)

At the cutoff by definition the unbounded optimum that generates indifference given $-k$'s true optumum is on the boundary is equal to the boundary, which yields

$$\bar{s}_{-k}\left(1 - \frac{z_{-k}(\bar{s}_{-k})}{x_{V-k}}\right) = (z_{-k}(\bar{s}_{-k}) - y_0)^2$$

Thus clearly the cutoff is also the point where the optimum if $-k$ is unboudned is exactly on the boundary. Solving yields:

$$\frac{\bar{s}_{-k}}{4x_{Vk}^2} = 1 - \frac{z_{-k}(\bar{s}_{-k})}{x_{V-k}}$$

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Clearly there is a unique solution $\bar{s}_{-k} > 0$, and it is straightforward to show that at this solution $z_{-k}\left(\bar{s}_{-k}\right)$ is strictly in between y_0 and $x_{V_{-k}}$. The exact solution is

$$\bar{s}_{-k} = \frac{4x_{V_k}^2\left(x_{V_{-k}} - y_0\right)}{x_{Vk} + 2\left(-x_{V_{-k}}\right)}$$

(Extracting the atom)

Finally, the value of the atom that generates the original indifference condition is simply

$$F_k\left(\underline{s}\right) = \frac{\bar{y}_{-k}\left(\underline{s}\right)}{x_{-k}/\alpha_{-k}}$$

From this we can make a few additional observations. First, if y_0 is closer to $-k$ than $\frac{x_{-k}}{\alpha_{-k}}$, then the indifference condition cannot be satisfied for any feasible value of the atom there is no way to induce entry by $-k$. Second, if $\frac{x_{-k}}{\alpha_{-k}}$ is closer to $-k$ than the same-sided veto player $x_{V_{-k}}$, then there is a feasible atom for any value of $\underline{s}$. Lastly, if $\frac{x_{-k}}{\alpha_{-k}}$ is in between y_0 and $x_{V_{-k}}$ then there is a maximum finite score such that the indifference condition could be satisfied.

B.5.2   Suppose $\underline{s}_k = 0$ (so $\underline{s}_{-k} = 0$)

So $\underset{s > \underline{s}_k}{supp}\{F_i\} \neq \emptyset$ for some i and $\underline{s}_k = \underline{s}_{-k} = 0$. We argue both have atoms at 0 and derive the values.

Values of both Atoms   We argue $Z_i'\left(0; F_{-i}\left(0\right)\right) = 0$ $\forall i$, implying

$$F_{-i}\left(0\right) = \alpha_i\left(\frac{z_i\left(0\right) - x_{V_{-i}}}{x_i - x_{V_{-i}}}\right) = \alpha_i\left(\frac{y_0 - x_{V_{-i}}}{x_i - x_{V_{-i}}}\right) \text{ and } F_{-i}\left(0\right)\frac{x_i}{\alpha_i} = x_i\left(\frac{y_0 - x_{V_{-i}}}{x_i - x_{V_{-i}}}\right)$$

Note that this requires $Z_i'\left(0; 1\right) > 0 \iff s_i^* > 0$ (both have strictly positive monopoly scores).

By previous results, support is a common nonempty mixing interval is $[0, \bar{s}]$ satisfying differential equations

$$f_{-i}\left(s\right)\left(V_i\left(s, y_i^*\left(s\right)\right) - V_i\left(s, y_{-i}^*\left(s\right)\right)\right) = C_i\left(s, F_{-i}\left(s\right)\right) \ \forall s \in [0, \bar{s}]$$

where

$$C_i\left(s, F_{-i}\left(s\right)\right) = \left(\alpha_i - F_{-i}\left(s\right)\right) - 2y_i'\left(s\right)\alpha_i\left(F_{-i}\left(s\right)\frac{x_i}{\alpha_i} - y_i\left(s\right)\right)$$

and is continuous in s and $F_{-i}\left(s\right)$. Note that at $s = 0$ we have $y_i\left(0\right) = z_i\left(0\right) = y_0$ (the only veto proof policy) so

$$C_i\left(0, F_{-i}\left(0\right)\right) = \left(\alpha_i - F_{-i}\left(s\right)\right) - 2y_i'\left(s\right)\alpha_i\left(F_{-i}\left(s\right)\frac{x_i}{\alpha_i} - y_0\right)$$

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We wish to show that $C_i(0, F_{-i}(0)) = 0$, which further implies $F_{-i}(s)\frac{x_i}{\alpha_i} \neq y_0$ and $y_i(s) = z_i(s)$ and $C_i(0, F_{-i}(0)) = -Z_i(s, F_{-i}(s))$ in a neighborhood around 0, yielding the result.

Suppose not, so there exists a proper solution to the system of differential equations and boundary conditions ($F_j(s) \geq 0$ and continuous $\forall s \in [0, \bar{s}]$ and $F_i(\bar{s}) = F_{-i}(\bar{s}) = 1$) such that $C_i(0, F_{-i}(0)) > 0$ (clearly cannot have $C_i(0, F_{-i}(0)) < 0$ which would $\to f_{-i}(0) < 0$.).

By continuity all around (a few minor details to fill in), $\exists \hat{s} > 0$ s.t. $C_i(\hat{s}, F_{-i}(\hat{s})) \geq$ some $q > 0$ $\forall s \in [0, \hat{s}]$. Now observe that for all $s \in [0, \hat{s}]$ we must have

$$f_{-i}(s) \cdot s \cdot x_i \left(\frac{1}{x_{V_i}} - \frac{1}{x_{V_{-i}}}\right) = f_{-i}(s) \cdot 2x_i (z_i(s) - z_{-i}(s))$$
$$\geq \quad f_{-i}(s) \cdot 2x_i (y_i(s) - y_{-i}(s)) = C_i(s, F_{-i}(s)) \geq q > 0$$

and therefore $f_{-i}(s) \geq \frac{z}{s}$ where $z = \dfrac{q}{x_i\left(\frac{1}{x_{V_i}} - \frac{1}{x_{V_{-i}}}\right)} > 0$.

Now consider the function

$$G_{-i}(s) = F_{-i}(\hat{s}) + z \log\left(\frac{s}{\hat{s}}\right)$$

This function satisfies $G_{-i}(0) = -\infty$, $G_{-i}(\hat{s}) = F_{-i}(\hat{s})$, and $f_{-i}(s) \geq \frac{z}{s} = g_{-i}(s)$ $\forall s \in [0, \hat{s}]$. But then we have $G_{-i}(s) - F_{-i}(s) = \int_s^{\hat{s}}(f_{-i}(s) - g_{-i}(s))\, ds \geq 0$ $\forall s \in [0, \hat{s}]$, so $F_{-i}(0) = -\infty$, contradicting that this is a proper solution.

Diff Eq Solutions in nonempty interval above $\underline{s} = 0$ From above, $Z_i'(0, F_{-i}(0)) = 0$ and $y_i(s) = z_i(s)$ and $C_i(s, F_{-i}(s)) = -Z_i(s, F_{-i}(s))$ $\forall i$ in a neighborhood above 0. Thus in this region the differential equations are

$$f_{-i}(s) \cdot 2x_i (z_i(s) - z_{-i}(s)) = -Z_i(s, F_{-i}(s))$$

or

$$f_{-i}(s) \cdot s \cdot x_i \left(\frac{1}{x_{V_i}} + \frac{1}{-x_{V_{-i}}}\right) = \alpha_i \left(1 + \frac{z_i(s)}{-x_{V-i}}\right) - F_{-i}(s) \cdot \left(1 + \frac{x_i}{-x_{V-i}}\right)$$

or

$$s \cdot f_{-i}(s)\frac{x_i}{\alpha_i}\left(\frac{1}{x_{V_i}} + \frac{1}{-x_{V_{-i}}}\right) = \left(1 + \frac{z_i(s)}{-x_{V-i}}\right) - F_{-i}(s) \cdot \frac{x_i}{\alpha_i}\left(\frac{1}{x_i} + \frac{1}{-x_{V-i}}\right)$$

or

$$s \cdot f_{-i}(s)\frac{x_i}{\alpha_i}\left(1 - \frac{x_{V_{-i}}}{x_{V_i}}\right) = (z_i(s) - x_{V-i}) - F_{-i}(s) \cdot \frac{x_i}{\alpha_i}\left(1 - \frac{x_{V-i}}{x_i}\right)$$

Now let $\bar{y}_i(s) = \frac{x_i}{\alpha_i}F_{-i}(s)$ denote i's *unconstrained optimum* at score s, which implies both $F_{-i}(s)$ (since $F_{-i}(s) = \frac{\bar{y}_i(s)}{x_i/\alpha_i}$) and $y_i(s)$ (since $y_i(s) = \min\{\max\{z_L(s), \bar{y}_i(s)\}, z_R(s)\}$).

Now the differential equations may be written in terms of this function as

$$s \cdot \bar{y}_i'(s)\left(1 - \frac{x_{V-i}}{x_{V_i}}\right) = sz_i'(s) + (z_i(0) - x_{V-i}) - \bar{y}_i(s)\left(1 - \frac{x_{V-i}}{x_i}\right)$$

We now conjecture a linear solution of the form $\bar{y}_i(s) = s\bar{y}_i' + \bar{y}_i(0)$, show such a solution exists, and explicitly derive it.

First, it is easily shown that our boundary condition $Z_i'(0; F_{-i}(0)) = 0$ also implies $(z_i(0) - x_{V-i}) - \bar{y}_i(0)\left(1 - \frac{x_{V-i}}{x_i}\right) = 0$. Rearranging yields:

$$\begin{aligned} \bar{y}_i(0) &= x_i\left(\frac{y_0 - x_{V-i}}{x_i - x_{V-i}}\right) \\ &= y_0 + (-x_{V-i})\left(\frac{x_i - y_0}{x_i - x_{V-i}}\right) \end{aligned}$$

Next, plugging in the linear form $\bar{y}_i(s) = s\bar{y}_i' + \bar{y}_i(0)$ into the differential equation yields

$$s \cdot \bar{y}_i'\left(1 - \frac{x_{V-i}}{x_{V_i}}\right) = sz_i'(s) - s\bar{y}_i'\left(1 - \frac{x_{V-i}}{x_i}\right) + (z_i(0) - x_{V-i}) - \bar{y}_i(0)\left(1 - \frac{x_{V-i}}{x_i}\right).$$

Then applying the crucial boundary condition yields:

$$s \cdot \bar{y}_i'\left(1 - \frac{x_{V-i}}{x_{V_i}}\right) = sz_i'(s) - s\bar{y}_i'\left(1 - \frac{x_{V-i}}{x_i}\right)$$

and therefore the linear solution is feasible. Cancelling all the s terms and rearranging yields:

$$\bar{y}_i' = \frac{z_i'(s)}{2 + (-x_{V-i}) \cdot \left(\frac{1}{x_{V_i}} + \frac{1}{x_i}\right)}.$$

Lastly, combining the preceding into a final solution yields:

$$\begin{aligned} \bar{y}_i(s) &= y_0 + (-x_{V-i})\left(\frac{x_i - y_0}{x_i - x_{V-i}}\right) + \frac{z_i'(s)}{2 + (-x_{V-i}) \cdot \left(\frac{1}{x_{V_i}} + \frac{1}{x_i}\right)} s \\ &= y_0 + (-x_{V-i})\left(\frac{x_i - y_0}{x_i - x_{V-i}}\right) + \frac{-x_{V_i}/x_{V-i}}{x_{V_i} + (-x_{V-i})\left(1 + \frac{x_{V_i}}{x_i}\right)\frac{1}{2}} \frac{s}{4} \end{aligned}$$

We can that this solution begins strictly closer to i than y_0, verifying that indeed $y_i(s) = z_i(s)$ $\forall i$ in a neighborhood around 0, and in this region this is indeed the solution. We can also see immediately that this solution has a strictly smaller slope than $z_i(s)$, implying that it must eventually cross $z_i(s)$. Beyond the score at which $\bar{y}_j(s)$ crosses $z_j(s)$ for some j we must have somebody off the boundary (for at least a nonempty interval).

Deriving cutoff where above solution no longer applies We derive the score $\bar{s}_i$ where $\bar{y}_i(\bar{s}_i) = z_i(\bar{s}_i)$; the above is the solution for $s \leq \min_j\{\bar{s}_j\}$ and not above this.

Using the above and $z_i'(s)s = z_i(s) - y_0$ we have

$$z_i(\bar{s}_i) - y_0 = (-x_{V-i})\left(\frac{x_i - y_0}{x_i - x_{V-i}}\right) + \frac{z_i(\bar{s}_i) - y_0}{2 + (-x_{V-i}) \cdot \left(\frac{1}{x_{V_i}} + \frac{1}{x_i}\right)}$$

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so

$$(z_i\left(\bar{s}_i\right) - y_0)\left(1 - \frac{1}{2 + \left(-x_{V_{-i}}\right)\cdot\left(\frac{1}{x_{V_i}} + \frac{1}{x_i}\right)}\right) = \left(-x_{V-i}\right)\left(\frac{x_i - y_0}{x_i - x_{V_{-i}}}\right)$$

and

$$\bar{s}_i\left(\frac{x_{V_i} + \left(-x_{V_{-i}}\right)\cdot\left(1 + \frac{x_{V_i}}{x_i}\right)}{2x_{V_i} + \left(-x_{V_{-i}}\right)\cdot\left(1 + \frac{x_{V_i}}{x_i}\right)}\right) = \frac{2x_{V-i}^2\left(x_i - y_0\right)}{x_i - x_{V_{-i}}}$$

implying

$$\bar{s}_i = \frac{4x_{V-i}^2\left(x_i - y_0\right)}{x_{V_i} + \left(1 + \frac{x_{V_i}}{x_i}\right)\left(-x_{V_{-i}}\right)} \cdot \frac{x_{V_i} + \left(1 + \frac{x_{V_i}}{x_i}\right)\frac{1}{2}\cdot\left(-x_{V_{-i}}\right)}{x_i + \left(-x_{V_{-i}}\right)}$$

Clear that if you set $x_i = x_{V_i}$ this is equal to the other $\hat{s}$ from the atom calcs, which is also weird!!

B.6   Remaining unamended proofs from previous version

B.6.1   Pure Strategy Equilibria

In the absence of veto players, pure strategy equilibria do not exist (Hirsch and Shotts (2015)). However, the presence of veto players potentially introduces such equilibria. As described in the main text, in any pure strategy equilibrium at least one entrepreneur $-k$ must be inactive, while the other must develop her monopoly policy (s_k^*, y_k^*). This will be an equilibrium when entrepreneur $-k$ is unwilling to pay the cost of defeating (s_k^*, y_k^*). Below we state the formal condition, using the notation $y_i^*\left(s_i; F_{-i}\left(s_i\right)\right)$ to refer to i's optimal score ideology to develop at score s_i given the probability $F_{-i}\left(s_i\right)$ that her opponent develops a lower-score policy.

Proposition 7 *In every pure strategy equilibrium at least one entrepreneur $-k$ must be inactive, and the other must develop policy (s_k^*, y_k^*). Let $\hat{s} = \max\left\{s_k^*, s_{-k}^*\right\}$. There exists a pure strategy equilibrium in which entrepreneur $-k$ is inactive i.f.f.*

$$\left(\hat{s} - s_k^*\right) + 2x_{-k}\left(y_{-k}^*\left(\hat{s}; 1\right) - y_k^*\right) \leq \alpha_{-k}\left(\hat{s} + \left[y_{-k}^*\left(\hat{s}; 1\right)\right]^2\right)$$

In the equilibrium, $F_{-k}\left(s_0\right) = 1$ and $F_k\left(s\right) = 0$ for $s_k < s_k^*$ and 1 otherwise.

Proof of Proposition 7

   As argued in the main text, at least one entrepreneur must be inactive in any pure strategy equilibrium. If $-k$ is inactive, then k develops her equilibrium policy (s_k^*, y_k^*) from the one entrepreneur game. This generates a utility for $-k$ producing a policy (s_{-k}, y_{-k}) with $s_{-k} \neq s_k^*$ of,

$$-\alpha_{-k}\left(s_{-k} + y_{-k}^2\right) + \mathbf{1}_{s_{-k} \geq s_k^*}\cdot V_{-k}\left(s_{-k}, y_{-k}\right) + \left(1 - \mathbf{1}_{s_{-k} \geq s_k^*}\right)\cdot V_{-k}\left(s_k^*, y_k^*\right)$$

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Scores $s_{-k} \in (s_0, s_k^*)$ are strictly dominated since they are costly and lose for sure. Scores $s_{-k} > s_k^*$ generate identical utility as the one entrepreneur game, and $-k$'s utility difference between developing the best policy $y_{-k}^* (s_{-k}; 1)$ for such a score and staying out of the contest is

$$(s_{-k} - s_k^*) + 2x_{-k} \left(y_{-k}^* (s_{-k}; 1) - y_k^* \right) - \alpha_{-k} \left(s_{-k} + \left[y_{-k}^* (s_{-k}; 1) \right]^2 \right) \tag{4}$$

From our analysis of the 1-entrepreneur game we know that when $s_{-k}^* > s_0$, the derivative of the objective function is > 0 for $s_{-k} < s_{-k}^*$, is $= 0$ for $s_{-k} = s_{-k}^*$, and is < 0 for $s_{-k} > s_{-k}^*$. If $s_{-k}^* = s_0$ then the derivative is < 0 everywhere.

Thus, if $s_{-k}^* > s_k^*$ then it is the strictly best score for $-k$ conditional on producing a winning score $> s_k^*$. Entrepreneur $-k$ then prefers to enter the contest and the pure strategy equilibrium does not exist if Equation (4) > 0 with s_{-k}^* substituted in. This is the condition in the statement.

If $s_{-k}^* \leq s_k^*$ then utility and Equation (4) are both decreasing over $s_{-k} > s_k^*$. Thus, if Equation (4) ≤ 0 with s_k^* substituted in, the equilibrium holds. Alternatively, if Equation (4) > 0 with s_k^* substituted in, then $-k$ can achieve a gain arbitrarily close to it by developing a policy with score $s_k^* + \varepsilon$ for sufficiently small ε, and the pure strategy equilibrium fails. ■

B.6.2 Mixed Strategy Equilibria

In mixed strategy equilibria of the model, both entrepreneurs are active with strictly positive probability, and mix over both the ideological locations and qualities of the policies they develop. In Hirsch and Shotts (2015) without veto players, we not only derive mixed strategy equilibria, but also show that they are unique. In the present draft we instead present sufficient conditions for mixed strategy equilibria, and omit claims of existence or uniqueness. However, we conjecture that both claims are effectively true, and they are work in progress for a future draft.

To state sufficient conditions for mixed strategy equilibria, we first introduce additional notation. The strategy profiles we will consider for the statement have the following two properties: (i) with probability 1 both entrepreneurs develop veto-proof policies of the form $(s_i, y_i^* (s_i))$, and (ii) the probability that both entrepreneurs develop veto-proof policies with the same score $s > s_0$ is 0; that is, there are no "score ties." Thus, in such profiles player i's expected utility from developing any veto-proof policy (s_i, y_i) can be written more precisely as,

$$\Pi_i^* (s_i, y_i; F) = -\alpha_i \left(s_i + y_i^2 \right) + F_{-i} (s_i) \cdot V_i (s_i, y_i) + \int_{s_i}^{\infty} V_i \left(s_{-i}, y_{-i}^* (s_{-i}) \right) dF_{-i} \tag{5}$$

In addition, her utility from developing the strictly best veto-proof policy with score s_i is $\Pi_i^* (s_i, y_i^* (s_i) ; F)$, which we henceforth denote as simply $\Pi_i^* (s_i; F)$. For the proposition, we use the notation x_{bVk} to represent the binding veto player opposite entrepreneur k, i.e., x_{Vl} for the entrepreneur at x_R and

x_{Vr} for the entrepreneur at x_L. And, as before s_i^* is the score of entrepreneur i's optimal proposal if she were a monopolist. We now characterize sufficient conditions for equilibrium.

Proposition 8 *A profile of strategies σ is a SPNE if there is an entrepreneur k and two thresholds $\underline{s}$ and $\bar{s}$ satisfying $s_0 < \underline{s} < s_k^* \leq \max\left\{s_{-k}^*, s_k^*\right\} < \bar{s}$ such that the following holds.*

(Policies) *With probability 1, both entrepreneurs $i \in \{L, R\}$ develop veto-proof policies of the form $(s_i, y_i^*(s_i))$.*

(Scores) *The equilibrium score CDFs (F_k, F_{-k}) satisfy the following conditions.*

1. *Entrepreneur k is always active, F_k has support $[\underline{s}, \bar{s}]$ with exactly one atom at $\underline{s}$, and*

$$F_k(\underline{s}) = \alpha_{-k}\left(\frac{\underline{s} + \left[y_{-k}^*(\underline{s})\right]^2}{2x_{-k}\left(y_{-k}^*(\underline{s}; F_k(\underline{s})) - z_k(\underline{s})\right)}\right) \iff \Pi_{-k}^*(s_0; F) = \Pi_{-k}^*(\underline{s}; F)$$

2. *Entrepreneur $-k$ is sometimes active, F_{-k} has support $s_0 \cup [\underline{s}, \bar{s}]$ with exactly one atom at s_0, and*

$$F_{-k}(\underline{s}) = \alpha_k \frac{|x_{bVk}| + 2|z_k(\underline{s})|}{|x_{bVk}| + 2|x_k|} \iff \left.\frac{\partial}{\partial s_k}\left(\Pi_k^*(s_k; F)\right)\right|_{\underline{s}} = 0$$

3. *For $s \in [\underline{s}, \bar{s}]$, the following coupled system of differential equations hold:*

$$\begin{aligned}\alpha_i - F_{-i}(s) &= f_{-i}(s) \cdot 2x_i\left(y_i^*(s) - y_{-i}^*(s)\right) \\ &+ 2\alpha_i \frac{\partial y_i^*(s)}{\partial s} \cdot \left(F_{-i}(s)\frac{x_i}{\alpha_i} - y_i^*(s)\right)\end{aligned} \qquad \forall i \in \{L, R\}.$$

Proof of Proposition 8

We proceed in two steps. First, we show that for $i \in \{L, R\}$, every possible policy (s, y) delivers utility $\leq \Pi_i^*(s_i, y_i^*(s_i); F)$ for some s_i. Second, we show that $\forall i \in \{L, R\}$, i's equilibrium utility Π_i^* is equal to $\max_{s_i}\left\{\Pi_i^*(s_i, y_i^*(s_i); F)\right\}$. These properties jointly imply that $i \in \{L, R\}$ has no profitable deviation and thus equilibrium.

Step 1

By Lemma 2, for any policy (s, y) that is not veto-proof or the status quo (which is the unique veto-proof policy with score s_0), entrepreneur i is weakly better off sitting out, i.e., $\Pi_i^*(s, y; F) \leq \Pi_i^*(s_0, y_0; F) = \Pi_i^*(s_0, y_i^*(s_0); F)$. Lemma 2 also implies that for any veto-proof policy (s, y) with a score $s > s_0$ where $-i$ has no atom, $\Pi_i^*(s, y; F) < \Pi_i^*(s, y_i^*(s); F)$. This takes care of all possible policies for the always-active entrepreneur k, since in the strategy profiles in Proposition 8 her opponent has no atoms above s_0.

It also takes care of almost all possible policies for the sometimes-inactive entrepreneur $-k$. However, we must also show that for $-k$, the payoff $\Pi_{-k}^*(\underline{s}, \hat{y}_{-k}; F)$ from developing any veto-proof

policy $(\underline{s}, \hat{y}_{-k})$ with $\hat{y}_{-k} \neq y_{-k}^*(\underline{s}; F_k(\underline{s}))$ at the score where k has an atom is weakly worse than the payoff $\Pi_{-k}^*\left(s_{-k}, y_{-k}^*(s_{-k}); F\right)$ from developing the optimal veto-proof policy at some score s_{-k}.

Let $w_k\left(y_k^*(\underline{s}), \hat{y}_{-k}\right)$ be the probability that k's policy is enacted when the entrepreneurs propose policies $(\underline{s}, y_k^*(\underline{s}))$ and $(\underline{s}, \hat{y}_{-k})$. Then $-k$'s utility from developing $(\underline{s}, \hat{y}_{-k})$ is

$$-\alpha_{-k}\left(\underline{s} + \hat{y}_{-k}^2\right) + F_k(\underline{s})\left[w_k\left(y_k^*(\underline{s}), \hat{y}_{-k}\right) V_{-k}\left(\underline{s}, y_k^*(\underline{s})\right) + (1 - w_k\left(y_k^*(\underline{s}), \hat{y}_{-k}\right)) V_{-k}\left(\underline{s}, \hat{y}_{-k}\right)\right]$$
$$+ \int_{\underline{s}}^{\infty} V_{-k}\left(s_k, y_k^*(s_k)\right) dF_k \tag{6}$$

Note that for $-k$ to prefer to develop $(\underline{s}, \hat{y}_{-k})$ rather than (s_0, y_0) requires $V_{-k}\left(\underline{s}, \hat{y}_{-k}\right) > V_{-k}\left(\underline{s}, y_k^*(\underline{s})\right)$ so Equation 6 $\leq$

$$-\alpha_{-k}\left(\underline{s} + \hat{y}_{-k}^2\right) + F_k(\underline{s}) V_{-k}\left(\underline{s}, \hat{y}_{-k}\right) + \int_{\underline{s}}^{\infty} V_{-k}\left(s_k, y_k^*(s_k)\right) dF_k. \tag{7}$$

But the argument for Lemma 2 implies that Equation 7 is strictly less than

$$\lim_{s_{-k} \to \underline{s}^+}\left\{\Pi_{-k}^*\left(s_{-k}, y_{-k}^*(s_{-k}); F\right)\right\} = -\alpha_{-k}\left(\underline{s} + y_{-k}^*(\underline{s})\right) + F_k(\underline{s}) V_{-k}\left(\underline{s}, y_{-k}^*(\underline{s})\right) + \int_{\underline{s}}^{\infty} V_{-k}\left(s_k, y_k^*(s_k)\right) dF_k.$$

Thus there must exist a score $\underline{s} + \epsilon$ such that $\Pi_{-k}^*\left(\underline{s} + \epsilon, y_{-k}^*(\underline{s} + \epsilon); F\right)$ is strictly greater than $-k$'s utility from developing $(\underline{s}, \hat{y}_{-k})$.

Step 2

We argue that for each entrepreneur i, equilibrium utility Π_i^* is equal to $\max_{s_i}\left\{\Pi_i^*(s_i, y_i^*(s_i); F)\right\}$. In the previous step we ruled out scores $s_i < s_0$. Also note that for any policy at a score $s_i > \bar{s}$, entrepreneur i is strictly better off developing $y_i^*(\bar{s}; 1)$, because $\bar{s} > s_i^*$, and as noted in our analysis of the monopoly model the entrepreneur's utility from enacting $(s_i, y_i^*(s_i))$ and having it enacted with probability 1 is strictly decreasing for $s_i > s_i^*$.

For entrepreneur $-k$, no score in $(s_0, \underline{s})$, can be optimal, because it would entail paying costs to develop a policy that loses for sure. And the proposition's first boundary condition, which specifies the size of k's atom at $\underline{s}$ ensures that $-k$ is indifferent between sitting out and developing a score at $\underline{s} + \epsilon$, i.e.,

$$0 = \lim_{s_{-k} \to \underline{s}^+}\left\{\Pi_{-k}^*\left(s_{-k}, y_{-k}^*(s_{-k}); F\right)\right\} - \Pi_{-k}^*\left(s_0, y_{-k}^*(s_0); F\right)$$
$$= F_k(\underline{s}) 2x_{-k}\left(y_{-k}^*(\underline{s}; F_k(\underline{s})) - z_k(\underline{s})\right) - \alpha_{-k}\left(\underline{s} + \left[y_{-k}^*(\underline{s})\right]^2\right).$$

For entrepreneur k, the proposition's second boundary condition ensures that $\Pi_k^*\left(\underline{s}, y_k^*(\underline{s}); F\right) > \Pi_k^*(s_k, y_k^*(s_k); F), \forall s_k < \underline{s}$, by specifying the size of $-k$'s atom at s_0. To derive the size of the atom, we first note that for $s_k \in [s_0, \underline{s}]$, a necessary condition for a policy $(s_k, y_k^*(s_k; F_{-k}))$ to maximize k's utility is that $y_k^*(s_k; F_{-k}) = z_k(s_k)$. Otherwise there exists a sufficiently small δ such that

$F_{-k}(s_k - \delta) = F_{-k}(s_k)$ and $F_{-k}(s_k) \cdot \frac{x_k}{\alpha_k} \in (z_L(s_k - \delta), z_R(s_k - \delta))$, and thus $y_k^*(s_k - \delta; F_{-k}) = y_k^*(s_k; F_{-k})$ which would mean that $\Pi_k^*(s_k; F) - \Pi_k^*(s_k - \delta; F) = -\alpha_k \delta$, i.e., $(s_k, y_k^*(s_k; F_{-k}))$ couldn't maximize k's utility. Next, we set $y_k^*(s_k; F_{-k}) = z_k(s_k)$ and differentiate Equation 5 to get

$$\frac{\partial}{\partial s_k}(\Pi_k^*(s_k; F)) = -\alpha_k \left(1 + \frac{\partial[z_k^*(s_k)]^2}{\partial s_k}\right) + F_{-k}(s_k)\left(1 + 2x_k\frac{\partial z_k^*(s_k)}{\partial s_k}\right). \tag{8}$$

For the equilibrium specified in Proposition 8, this must be ≥ 0 for $s_k < \underline{s}$. In fact it must be equal to 0 at $\underline{s}$. To see why, note that for any s_k where F_{-k} is continuous,

$$-\alpha_k\left(1 + \frac{\partial[z_k^*(s_k)]^2}{\partial s_k}\right) + F_{-k}(s_k)\left(1 + 2x_k\frac{\partial z_k^*(s_k)}{\partial s_k}\right)$$

$$\leq -\alpha_k\left(1 + \frac{\partial\left([y_k^*(s_k; F_{-k})]^2\right)}{\partial s_k}\right) + F_{-k}(s_k)\left(1 + 2x_k\frac{\partial y_k^*(s_k; F_{-k})}{\partial s_k}\right)$$

by Lemma 2, and $f_{-k}(s_k) \cdot 2x_k\left(y_k^*(s_k; F_{-k}) - y_{-k}^*(s_k; F_k)\right) \geq 0$, so the differential equation in the proposition's third condition for $\underline{s} + \varepsilon$ with ε sufficiently small could not be satisfied unless the boundary condition holds with equality. The boundary condition is then derived by setting (8) equal to 0 at $\underline{s}$, plugging in for $z_k^*(s_k)$ and $\frac{\partial z_k^*(s_k)}{\partial s_k}$, using Definition 1 in the main text to get $F_{-k}(\underline{s}) = \alpha_k\frac{|x_{bVk}| + 2|z_k(\underline{s})|}{|x_{bVk}| + 2|x_k|}$.

The coupled differential equations in the proposition's third condition are derived by differentiating Equation 5 for each entrepreneur and setting it equal to zero to ensure that her payoff $\Pi_i^*(s_i, y_i^*(s_i); F)$ is constant on the interval $[\underline{s}, \overline{s}]$ where the entrepreneurs mix continuously. Specifically, for each $i \in \{L, R\}$

$$0 = \frac{\partial\Pi_i^*(s, y_i^*(s); F)}{\partial s}$$

$$= -\alpha_i - 2\alpha_i y_i^*(s)\frac{\partial y_i^*(s)}{\partial s} + F_{-i}(s)\left[1 + 2x_i\frac{\partial y_i^*(s)}{\partial s}\right] + f_{-i}(s) \cdot 2x_i\left(y_i^*(s) - y_{-i}^*(s)\right)$$

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i\left(y_i^*(s) - y_{-i}^*(s)\right) + 2\alpha_i\frac{\partial y_i^*(s)}{\partial s} \cdot \left(F_{-i}(s)\frac{x_i}{\alpha_i} - y_i^*(s)\right).$$

∎

Intuition and Computational Procedure Deriving equilibria satisfying the conditions in Proposition 8 is cumbersome to do analytically, but straightforward to do numerically. The differential equations and boundary conditions that define the equilibrium score CDFs (F_k, F_{-k}) can be intuitively understood by considering the incentives of each entrepreneur. First, the entrepreneur k who is always active knows that her competition will develop no policy (i.e., propose the status quo) with probability $F_{-k}(s_0) > 0$. Thus, increasing her score over the interval $[s_0, \underline{s}]$ will not generate any

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benefits in the form of increasing the chance of winning the contest. She must therefore *actively prefer* to develop a policy with score $\underline{s} > s_0$ over policies with lower scores that will win the contest with the same probability. This generates the boundary condition on $F_{-k}(\underline{s})$. Second, the entrepreneur $-k$ who is sometimes inactive must be exactly indifferent between staying out of the policy contest, and entering the contest with a policy at score $\underline{s} > s_0$. This policy has strictly positive quality, and therefore a strictly positive up-front cost to develop. She must then also have a strictly positive probability $F_k(\underline{s})$ of winning the contest with it, which generates the second boundary condition.

The differential equations in Proposition 8 arise from the fact that both entrepreneurs must be indifferent over developing all ideologically optimal veto-proof policies with scores in the common support interval $[\underline{s}, \overline{s}]$. Note that both differential equations contain both score CDFs F_k and F_{-k}, a complication that arises from the partial dependence of each entrepreneur's optimal ideologies $y_i^*(s_i)$ on her opponent's score CDF F_{-i}. Nevertheless, the incentives described by the differential equations are intuitive. The left hand side represents the marginal cost of producing a policy with a higher score given a *fixed* probability $F_{-i}(s)$ of winning the contest. The two terms in the right hand side represent the marginal benefit of producing a policy with a higher score, which is two-fold. First (and as in Hirsch and Shotts (2015) it increases the probability of victory by $f_{-i}(s)$, which results in a beneficial change in ideological outcomes from $y_{-i}^*(s)$ to $y_i^*(s)$. Second, if policy is constrained by an opposing veto player ($F_{-i}(s)\frac{x_i}{\alpha_i} \neq y_i^*(s)$), then there is an additional benefit of moving policy closer to the unbounded optimum.

To characterize equilibria, we proceed as follows. For each set of parameter values, we first verify whether pure strategy equilibria exist by checking the conditions in Proposition 7. Then, for parameters such that a pure strategy equilibrium does not exist we compute mixed strategy equilibria as follows. We conjecture an entrepreneur k who is always active and then search over candidate values of $\underline{s} \in [s_0, s_k^*]$ to support a mixed strategy equilibrium. An equilibrium is identified when the score CDFs satisfying the boundary conditions at the candidate $\underline{s}$ and the pair of coupled differential equations also satisfy the required boundary condition $F_{-k}(\bar{s}) = F_k(\bar{s}) = 1$ at some $\bar{s}$ (this boundary condition is implicit in the statement of equilibrium because the support of the CDFs is common and atomless above $\underline{s}$). In all the parameter profiles we have considered for which a pure strategy equilibrium does not exist, there exists exactly one mixed strategy equilibrium that satisfies the sufficient conditions in Proposition 8.