# Distorted Democracies 

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#### Abstract

Malapportionment is the discrepancy between the relative allocation of seats and the relative allocation of votes across districts. It is a global phenomenon that varies substantially across nations. This paper represents a systematic attempt to evaluate its impact on the ideological bias of legislatures and establish its relative importance relative to other institutional sources of political bias among democracies. To that end we put together a new comprehensive data set of vote allocation, voter preferences, and party ideology at the national and subnational levels for a large percentage of the world's democracies. We uncover two findings. First, contrary to extant priors, the direction of the ideological impact of malapportionment is heterogeneous across democracies, especially in lower houses. Second, the distortionary effect of malapportionment is meaningful but varied, and in many cases far from dominant when compared to other sources of bias ${ }_{-}^{1}$


[^0]
## 1 Introduction

Even though democratic elections represent, at least ideally, the interests of citizens on an equal footing, modern parliaments are hardly mirrors of society. Certain individuals and territories often enjoy a higher degree of influence than their relative demographic weight of the polity would imply. As a result, the distribution of preferences of policymakers is not always congruent with the distribution of preferences among citizens, leading to a violation or, at least, a distortion of the principle of equal representation.

This bias in representation (and the policy distortions that follow from it) has many sources: the structure of the legislature and, particularly, the overrepresentation (in most federal systems) of specific subnational units in the upper house; the type of electoral rules, mostly driven by the distinction between single member district (SMD) and multi-member districts (MMD); the magnitude of electoral constituencies; and, directly related to the size the district, the degree of legislative malapportionment, that is, the discrepancy between the relative share of seats and voters across districts.

Our understanding of the distortionary effects of each of these channels is imbalanced. A very rich literature shows that district magnitude, as well as other mechanisms employed to translate votes into seats, shape voters' strategic choices (Kedar 2005), the strategies of coordination within and between parties (Cox, 1997), government formation (Laver, 1998), and, as a result, policy outcomes. While single-member-district (SMD) systems in unidimensional policy spaces tend to privilege the median voter within the demos, proportional representation (PR) turns the median parliamentarian into the kingmaker Austen-Smith, 2000; Huber and Powell, 1994, Powell and Powell Jr, 2000. Insofar as the median parliamentarian is more to the left than the median voter, PR works as a force in favor of income redistribution. PR has thus been linked to the more frequent formation of center-left coalitions and larger welfare states Iversen and Soskice 2006, while SMD elections have been founded to be biased against redistribution when left-wing voters are geographically concentrated (Rodden, 2019).

By contrast, we know relatively little about malapportionment and its effects on the representation of voters' preferences. Indeed, the study of the ideological impact of malapportionment has been limited to a few case studies and comparative papers relying on small (and generally unrepresentative) samples of countries. See Rydon (1968); May (1974); Yamakawa (1984); Jackman (1994); Baker (1995). As shown in Figure 1, which displays the level of malapportionment in 65 countries in the world ( 13 countries for the upper house data), malapportionment is a global phenomenon that varies substantially across nation, ranging from almost perfectly apportioned districts in countries like Latvia
and South Africa, to nations such as Argentina, where several ovrerrepresented districts elect twice as many deputies as the median district in the legislature ${ }^{2}$ For lower house elections, the mean and the standard deviation of malapportionment are 0.06 and 0.04 , respectively, with an interquartile range of 0.04 and a skewness equal to 1.59. For upper house elections, the mean and standard deviation are 0.26 and 0.14 respectively, with an interquartile range of 0.17 and skewness equal to -0.56 . Overall, malapportionment is higher in upper than in lower chambers. In addition, lower chambers are more malapportioned in bicameral than unicameral systems on average. ${ }^{3}$

Figure 1: Malapportionment in the World


Note: The figure displays the level of malapportionment in lower (blue) and upper (red) house elections according to Samuels and Snyder (2001).

This paper offers a comprehensive study of the size and the direction of biases associated with legislative malapportionment. By size we refer to the relative importance of malapportionment as a share of the

[^1]overall bias introduced by representative institutions.
By direction we refer to the identity of the party family that in any given polity at any given time benefits from the "voter representation bias" associated with malapportionment. The dominant position in the current literature is that malapportionment always benefits conservative forces. The idea is simple: elites design and exploit malapportionment to preserve their economic and political advantage. Indeed, malapportionment has been shown to deter redistribution in late industrializers with weak welfare states, or to maximize the extraction capacity of overrepresented members in federations. For instance, Bruhn et al. (2010) show that overrepresented districts in Latin America are dominated by parties aligned with the elite. Beramendi et al. (2017) argue that representation bias caused by malapportionment leads to the underrepresentation of constituencies that prefer progressive taxes in the national parliament. Using data from Argentina, Chile and Mexico, the authors find that the higher the level of malapportionment, the more limited are efforts to reduce income inequality. Similarly, Ardanaz and Scartascini (2013) show that legislative malapportionment may prevent the use of personal income taxes as a major revenue source by skewing the distribution of political power across groups. By and large, the posited mechanism across these contributions is similar: the type of elite that gets to condition the legislative agenda due to overrepresentation managed to extract a surplus of budgetary rents for herself or her constituents. ${ }^{4}$ Yet, no matter how compelling this logic may be in the context of late developing federations with large levels of spatial inequalities, there is little reason to believe that it constitutes a universal law.

To estimate the size and direction of malapportionment, we develop two novel measures: "total representation bias" or the indicator of the overall bias associated with the system of representation; and "voter representation bias," which estimates the extent to which malapportionment - net of the effect of other electoral institutions - distorts the representation of voters' preferences. We then employ both measures to calculate the relative bias malapportionment introduces in a legislature, that is, the distortion of representation due to malapportionment relative to other institutional features (district magnitude, electoral formulae, etc.).

To determine the relative directional impact and size of malapportionment, we rely on two types of measurement exercises. In the first place, we collect national and district level indicators of malap-

[^2]portionment, along with the ideological stances of voters, and electoral data on party support within and across districts from over 19,000 electoral districts in 65 democratic countries and 247 legislative terms in both lower and upper house elections. In the second place, we employ individual-level data from Spanish surveys. The latter allows us to test the robustness of our findings as well as to engage in a number of district-level simulations to estimate the effect of the different components of the electoral system as well as of the levels of malapportionment on the distortion of representation: using simulations, we can approximate the size and direction of the bias associated with malapportionment in a way less constrained by multiple sources of unobserved heterogeneity. Spain is an ideal case for this exercise because of its complexity in terms of the varying magnitude of its electoral districts, the use of thresholds, and its mid-range levels of malapportionment.

After mapping out a high degree of heterogeneity in both the direction and the size of the bias associated with malapportionment around the world, we establish two main findings. In the first place, and contrary to existing literature claiming that malapportionment carries a right wing bias, we find that district overrepresentation is in fact positively correlated with the share of left-wing voters in almost half of our legislative bodies. At the end of the day, malapportionment' ideological bias depends on the territorial distribution of left and right-wing voters. In rural Mexico or Peru, the less populated (and politically overrepresented) regions are left-wing strongholds. In turn, some cities, which tend to be underrepresented in seats, vote for conservative parties - Madrid, in Spain, is a case in point. This finding has important implications for our understanding of the role of systems of representation in political economy and the formation of welfare states.

In the second place, the level of representation distortion generated by legislative malapportionment tends to be small, on its own and relative to the effect of other electoral institutions. Among lower houses, malapportionment only explains one tenth of the total difference between voters' preferences unmediated by elections and the preferences of parliamentarians in more than one half of our observations, and it only accounts for more than half of all distortions in one out of ten cases or countryelections. Among upper houses, the role of malapportionment is stronger. It explains more than 10 percent of the total distortion between votes and seats in three fourths of our observations.

The paper is organized as follows. Section2discusses the potential sources of representative bias in democracies, describes our data, and introduces our novel measures of representation bias. Section 3 examines the direction of the impact of malapportionment, finding that its ideological bias varies with country. Section 4 reports the estimated size of the bias of malapportionment in relation to other
institutional features of democracy - employing cross-national evidence as well as a set of simulations for one country and individual-level data of voters' preferences. Finally, section 5 summarizes our findings and lays a path for future research on the biases of representative institutions.

## 2 Understanding Malapportionment and its Effects: Scope and Biases

The first goal in this paper is to establish whether, where, and how malapportionment is a source of ideological bias across representative democracies. This requires, in turn, placing its role within a more general framework of the potential sources of representation bias coming from two kinds of mechanisms: behavioral and institutional.

The behavioral causes of representation bias are of three kinds: elite selection, turnout differentials, and spatial clustering of voters. First, the electoral competitive process to select political elites biases the final composition of legislatures and governments by rewarding those individuals with money, connections, and personal resources (Manin 1997. Dal Bó et al. 2017). Second, social groups that systematically abstain more than the average end up being underrepresented in parliament Wolfinger and Rosenstone, 1980 Grofman, Koetzle and Brunell 1997). These tend, in many nations, to be lower income individuals and thus potential constituents of left-wing economic programs. Finally, the geographical concentration of particular types of voters, particularly left-wing voters in advanced industrial democracies, tend to distort their political representation (Gudgin and Taylor 1979 Johnston 1981: Taylor, Gudgin and Johnston. 1986. Rodden. 2019).

The composition of the electorate across existing and potential districts also opens up the possibility of an institutional manipulation of the process of preference aggregation. First, the electoral rules employed to translate votes into seats affect the congruence between voters' and legislators' policy positions through two channels: mechanically, because votes have to be translated into a smaller quantity, seats; and, for strategic reasons, by inducing voters to eschew voting for small, "unviable" parties that may represent their preferences more closely (Duverger 1954 Laakso and Taagepera, 1979; Cox 1997. Taagepera 2007. Second, the allocation of seats to each district can bias political representation. As already described, a substantial majority of countries do not assign seats strictly in proportion to the population of each district. The distortions associated with malapportionment are both direct (or mechanical) and strategic: as the number of assigned seats changes, so do parties and even voters' incentives to behave strategically at the polls. The relative importance of malapportionment as a source of bias rests on the magnitude of these two effects.

### 2.1 Measurement and Analytical Strategy

To estimate the impact of malapportionment on representational bias, we develop two novel measures:

1. Total Representation Bias (TRB): a measure of the overall distortion in ideological representation caused by all the factors of the representation system.
2. Voter Representation Bias (VRB): a measure of the ideological distortions caused exclusively by malapportionment.

In this section, we describe first the baseline data that we employ to calculate our two key measures, namely voter ideology and malapportionment. Subsequently, we present the technical definitions of both VRB and TRB.

### 2.1.1 Baseline Data

Our analysis requires two types of data: the ideology of voters, to evaluate how different aspects of the representation system process them, and measures of malapportionment. After defining democracy as any country-year with a score of six or above in Polity IV at the time of the election of interest (Marshall, Jaggers and Gurr, 2011), we have generated a new data set at the constituency level for all available elections in every democratic country in the world. It contains the following variables: population entitled to vote at the district and at the national level; district magnitude; total number of parliamentary seats elected; share of votes by party; and, finally, the number of seats each party has obtained at the district level. The scope of this data collection greatly exceeds existing efforts to measure representation bias.

The attribution of ideology to voters is critical for the calculation of the representation bias. We derive the ideology of voters by using the economic policy position of the party they voted for, as coded in expert surveys - for doing so we use the Chapel Hill Experts Survey (Ryan et al. 2020) [CHES], the Political Representation, Executives, and Political Parties Survey Wiesehomeier, Singer and Ruth-Lovell, 2019) [PREPPS], the V-Dem Party Survey (Lührmann et al. 2020) [V-Dem] and the Global Party Survey (Norris, 2020) [GPS]; see Appendix Afor further details on the use of expert surveys to derive party's ideological stances. The underlying assumption behind using party data for deriving individual-level political stances is that a voter's ideology is close to the party's they vote for. We assume that all voters from a party share the same ideology, which matches with the party's ideology they have voted for.

This is, of course, a forced empirical constraint given the lack of (comparable) individual-level data across countries, but in section 3.3 we relax this assumption by using individual-level data in Spain to calculate the district-level preference structure and the extent to which malapportionment causes distortion in representation.

We focus on a single ideological dimension - the preferences of voters toward the economy and economic policy - for three reasons. First, economic ideology travels easily across countries, regardless of their institutional and social contexts. By contrast, measures of cultural ideology tend to be more idiosyncratic to each country and may be distorted by the latter's internal structure of competition. Second, a measure of economic ideology allows us to identify preferences for redistribution and, hence, deviations in left-right representation bias, which can be directly linked to the literature on welfare states and redistribution that has played a central role in many studies of malapportionment. Finally, the wording of the economic ideology variable is practically identical in three out of the four surveys we employ, allowing for direct comparisons.

To build the data set, we mainly rely on the information on lower and upper house elections reported in the Constituency-Level Electoral Archive (CLEA) Kollman et al. 2019) since 1995. For those countries and elections where data from the CLEA is missing or incomplete, we retrieve information provided by national statistical offices and/or national electoral commissions as well as other election repositories or newspapers. For a few cases, we have also determined the seats elected at the district level by applying the country's electoral law. Finally, we have engaged in multiple data checks to make sure that the sum of the percentage of votes cast in each electoral constituency is equal to 1.5 Our final data set contains over 19,000 constituency-level observations (almost 200,000 party-level observations) from 189 elections in lower and 58 elections in upper house elections in 65 countries in the world ( 13 countries for the upper house data) ${ }^{6}$

### 2.1.2 Measuring Malapportionment

Figure 1 reports the level of national malapportionment. To understand its relationship with the policy preferences of voters and compute our measures of VRB and TRB, however, we need a disaggregated

[^3]measure of malapportionment at the district level. We calculate legislative malapportionment at the district level or $M A L_{\text {dist }}$ by employing the (log) RRI or the "Relative Representation Index", developed by Ansolabehere et al. $(2002)^{7}$.
$$
M A L_{d i s t}=1+\log R R I
$$
where:
$$
R R I=\left(\frac{\frac{\text { Districtseats }}{\text { Districtpopulation }}}{\frac{\text { Totalseats }}{\text { Totalpopulation }}}\right)
$$

RRI equals 1 when the constituency is perfectly apportioned according to population. A value higher (lower) than 1 indicates that the district is overrepresented (underrepresented). Prior to the logtransformation of the variable, a value of 2 implies that a given district elects twice as many deputies as it should according to the registered population. In turn, a value of 0.5 reveals that the district is electing only half of the deputies it should given its population. Due to the unbounded character of the variable for values above unity and to obtain a more normal distribution, we log-transform the measure of the RRI. We then add 1 to the log value of RRI to preserve the value of 1 as the reference for a perfectly-apportioned district. The log-transformation of RRI results in a symmetric distribution of under and overrepresented districts. For instance, a value of 1.3 (0.7) represents a district electing twice (half) the deputies it should ${ }^{9}$

### 2.2 Voter Representation Bias

As introduced above, Voter Representation Bias (VRB) is the total share of bias that is triggered by malapportionment, net of the other electoral institutions. Formally, it is defined as follows:

$$
V R B=I V S\left[\text { mean }\left(\text { Voter Ideolog } y_{\% \text { seats }}\right)\right]-I V P\left[\text { mean }\left(\text { Voter Ideolog } y_{\% \text { population }}\right)\right]
$$

where IVP or Ideology of Mean Voter according to Population corresponds to the policy preferences (along an economic ideological continuum) of the mean voter in the whole country when excluding

[^4]the effect of malapportionment. To determine IVP, we simply estimate the ideological mean of all voters at the national level.

In turn, IVS or Ideology of Mean Voter according to Seats consists in the policy preferences of the mean voter factoring in the effect of malapportionment. To calculate it, we compute the mean of voters' preferences after weighing voters by the degree of over- or underrepresentation of the district in which they live ${ }^{10}$

Table 1 provides an example that should clarify the procedure to calculate both IVS and IVP. Consider a country with 1,000 voters allocating 100 deputies between two districts, A and B , with each district receiving 50 seats. Although both districts have the same number of seats, district A comprises $75 \%$ of the country's population entitled to vote. B holds $25 \%$ of it. In district $\mathrm{A}, 60 \%$ of the voters have an ideology of 4 and $40 \%$ of 6 . In district B, the proportions are reversed: $40 \%$ of the votes have an ideology of 4 and $60 \%$ of 6 . Table 1 sorts individuals in a country, first, by ideology, and second by district. It shows that 550 individuals have an ideology of 4 (in a continuum from 0 to 10): 450 individuals from district A, 100 from B. The rest place themselves at 6: of these, 300 are living in district $A$ and 150 in $B$, thus making 450 additional voters. The mean voter in the country is 4.9.

Table 1: Malapportionment and Representation Bias

|  |  |  | \%Vote | Pop. | Seats | District | Unweigh. Weighted Weighted |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| District | Voters | Ideology | district district district Ratio Mean Voters Mean |  |  |  |  |  |  |
| A | 450 | 4 | $60 \%$ | 750 | 50 | $1 / 3$ | 150 |  |  |
| B | 100 | 4 | $40 \%$ | 250 | 50 | 1 | 100 |  |  |
| A | 300 | 6 | $40 \%$ | 750 | 50 | $1 / 3$ | 100 |  |  |
| B | 150 | 6 | $60 \%$ | 250 | 50 | 1 |  | 150 |  |
|  |  |  |  |  |  |  | 4.9 |  | 5 |

Malapportionment alters the political weight of each district to the point that a vote in district B is three times more consequential to determine policy than a vote in district A . Hence, to adjust for malapportionment, we multiply the votes of district A by the seat/voter ratio of district A with respect to district B or $1 / 3$ in this particular case. The ideal policy position of the weighted average voter is now

[^5]5. In this particular example, the ideological distortion of representation due to malapportionment equals the difference between the weighted mean (IVS of 5) and the unweighted mean (IVP of 4.9) or +0.1 .

Notice that the level of VRB, which measures where the mean voter is placed in the population vis-à-vis where it is placed in the national parliament, is independent from the so-called mechanical effects of the electoral laws (i.e., district magnitude, the presence of a national or regional legal threshold or the type of ballot, among others). However, it may still be affected by the psychological or strategic effects induced by electoral rules. In particular, despite the fact that VRB is calculated before votes are translated into seats, in an observational setting we cannot rule out the fact that voters cast their ballots strategically by anticipating the mechanical effects of electoral laws. We come back to this question in subsection 4.2

In our calculations, we employ the mean of the district rather than the median of the district for the following reason. Under any voting method that satisfies the Condorcet criterion, the winner will be the candidate preferred by the median voter (Black 1948 Downs 1957). As a result, changes in the median ideology of the district - instead of the mean - should be the ones consequential for representation. While this is still the case in our argument, given our empirical strategy, the use of a median voter faces the following problem. Because we use the ideological position of parties (instead of voters) to assess the existence of representation bias, relying on the median ideology would underreport the change in policy positions of legislators with respect to voters under some political scenarios. More specifically, when the number of parties competing at the national level is low and/or when the distribution of party votes in the country is such that an electorally successful party is placed around the median of the electoral spectrum, the median voter according to the share of votes and the median voter according to the share of seats could coincide even when malapportionment is very high.

To clarify this point, consider the three following hypothetical cases. First, in a country with two parties, A and B, receiving $60 \%$ and $40 \%$ of the votes, respectively, the median voter is placed in A. For malapportionment to change the median voter, the electoral support for party B would have to be extremely concentrated in overrepresented districts to offset the distance of 10 percentage points between the $40 \%$ of the party vote and the $50 \%$ where the median voter is placed. Second, consider a country with three parties, A, B and C, each of them receiving one third of the votes and where B was a center party receiving all the votes cast by individuals placed between the percentile 33 and the percentile 66 in a continuous ideological space. Malapportionment would be unlikely to change the
median parliamentarian. Finally, consider a third country with two parties, A and B, receiving 49.9\% and $50.1 \%$ of the votes, respectively. Even though the median voter is placed in B, a very small degree of malapportionment could be enough to flip the median voter from B to A. To avoid these swings, we calculate the VRB according to the mean value of ideology. Still, in Section 3.3 and in Appendix Flwe test the robustness of our evidence by employing individual-level data to calculate the median VRB.

### 2.3 Total Representation Bias

The Total Representation Bias (TRB) measures the overall distortion in ideological representation caused by all the factors of the representation system. In empirical terms this is the total deviation in political representation between the mean voter and the mean parliamentarian in a country. We calculate it by subtracting the mean party's ideology according to the share of seats in the national parliament (IPS) - note that in the VRB it was the voter's ideology instead of the party's one - from the mean voter's ideology according to the share of population (IVP). We continue to use the mean deviation in the total representation rather than the median. Formally, TRB is defined as follows:

$$
T R B=\mid[I P S[\text { mean(Party Ideology\% seats })]-I V P[\text { mean(Voter Ideology\% population })] \mid]
$$

TRB differs from VRB in two important ways. First, it is comprehensive, that is, it considers jointly all potential sources of bias. Second, it is non-directional, that is, it captures the overall incidence of bias without discriminating which ideological families benefit from it.

Figure 2 (dark columns) displays the total representation bias by country in lower (panel left) and upper (panel right) house elections. In lower chambers, almost one fourth of all elections register a TRB above 0.2. TRB is above 0.5 in 8 percent of the cases. In upper house elections, TRB is much higher: above 0.2 in 54 percent of the elections, over 0.5 in 23 percent of the elections, and above 1 in 7 percent of the cases.

## 3 Direction: Who Benefits from Malapportionment?

We now turn to assess whether, as claimed by an incipient literature on representation and the welfare state, the extent and direction of malapportionment correlate with the structure of voters' preferences at either the national or district level. In subsection 3.1we examine the patterns of malapportionment

Figure 2: Total Representation Bias by Country


Note: Dark bars in the figure show the mean (of the different elections) total representation bias in lower and upper chambers, by country.
and their correlation with the preferences of voters. In subsection 3.2 we assess whether these patterns are consequential for the emergence of a representation bias. Finally, subsection 3.3 uses individuallevel data from Spain to gauge the extent to which the evidence found in the two previous sections is robust to the use of survey data of voter preferences.

### 3.1 Malapportioned Districts and Their Preference Structure

To determine the association between malapportionment and district-level ideology, the dark bars in Figure 3 report the coefficient of correlation for each country between district malapportionment (measured through (log)RRI) and the mean economic ideology in each district. The value of malapportionment includes all elections for which we have data in each country. A negative coefficient implies that left-wing districts are more overrepresented than right-wing districts. The highest negative correlation is Mauritius, with a coefficient close to -0.5 . A positive coefficient implies overrepresentation of the right. The extreme case is Macedonia, with a positive correlation coefficient close to 0.7 . The left
panel plots values for lower chambers; the right panel does the same for upper chambers.

In almost half of our countries, district overrepresentation rises with left-wing electorates, contradicting the consensus in the existing literature that claims that malapportionment benefits conservative districts. The correlation is particularly strong in countries as different as Belgium, Georgia, Iceland, India, Mauritius, Peru, Switzerland, Georgia, Iceland and the United Kingdom. By contrast, in cases such as the Czech Republic, Ecuador, Macedonia, Spain and Uruguay, malapportionment is associated with conservative electorates. In between, we find several countries, such as Chile, Italy, South Africa or South Korea, where ideology is not clearly related to malapportionment. A similar pattern takes place in upper houses. Malapportionment and left-wing districts are positively correlated in Australia, Chile, and Poland but negatively correlated in Bolivia, Japan, Spain, or the US. Interestingly, Brazil, a case of high malapportionment, does not have a strong ideological bias in its extremely malapportioned senate. To the extent that it shows a bias, it is toward the left in the Brazilian Senate. Figure D. 1 in Appendix $D$ displays a series of two-way scatter-plots between the ideology and the (log)RRI by country.

Figure 3 also graphs, using hollow bars, the country-election mean of national malapportionment according to the formula developed by Samuels and Snyder (2001). Across countries, the national average of malapportionment and the correlation coefficients are uncorrelated, suggesting that the strength of the correlation is not necessarily related to how big is malapportionment in each country.

### 3.2 Does Malapportionment Create Representation Bias?

The correlation between ideology and the over- or under-representation of seats only matters for representation bias when malapportionment is big enough to trigger distortions in representation. Accordingly, we turn now to assess the extent to which malapportionment is able to trigger representation bias in different countries.

Figure 4 shows the histogram of the VRB in 189 elections in lower (blue) and 58 elections in upper (red) house elections in 65 countries in the world ( 13 countries for the upper house data). The value in the $x$ axis can be directly interpreted as the difference between the mean voter in the national legislature and the mean voter in the country. For instance, a value of 0.1 reveals that the mean voter in the national parliament is 0.1 (in a 0 to 10 scale) to the right of the country's mean voter - as in the example shown in Table 1. For a majority of nations, we cannot assume a left- or right-wing bias of malapportionment in any particular country case. The distortion of representation in lower house elections is mostly

Figure 3: Correlation between (log)RRI and Ideology


Note: Blue and red horizontal bars in the figure display the country-election coefficient of correlation between the (log) Relative Representation Index and the district's mean ideology in lower (panel left) and upper (panel right) house elections. Positive values reveal higher levels of malapportionment. Hollow bars show the (country-election mean) of the level of malapportionment in each country. Values in the x-axis stand for both the coefficient of correlation and for the level of malapportionment.
within a $\pm 0.1$ bias - and evenly distributed both to the left and to the right of the ideological spectrum for our sample of countries. By contrast, in upper house elections the VRB is higher and markedly overrepresents right-wing districts. Relying, as much of the existing empirical literature does, on lower chamber malapportionment as the country-level measure of malapportionment has resulted in underplaying the degree of right-wing bias introduced by upper legislative chambers and, therefore, mischaracterizing the overall ideological bias induced by legislative malapportionment (Ardanaz and Scartascini 2013, Boone and Wahman, 2015, Bruhn, Gallego and Onorato, 2010; Ong, Kasuya and Mori 2017).

Figure5turns to display the (average) Voter Representation Bias for each country separately. Negative (positive) values indicate that malapportionment generates a leftist (rightist) representation bias in the voter's ideology. In line with the evidence displayed in Figure 3 . Figure 5 shows that malapportionment

Figure 4: Histogram of the VRB in Lower and Upper Chambers


Note: The figure displays the distribution of the mean of the Voters' Representation Bias (VRB) in lower (blue) and upper (red) house elections. The vertical line at the zero value implies no representation bias attributable to malapportionment. The note below the legend displays the mean and standard deviation (between brackets) of the VRB for lower and upper chamber elections.
causes a representation bias in some countries, yet the direction of the effect is far from homogeneous, particularly in lower house elections (left graph). Indeed, in lower chambers the voter's representation bias is just as likely to benefit the left and the right across our sample of countries. The magnitude of the impact is modest in most countries. The mean across different elections only exceeds 0.02 in 9 countries and less than one fifth of all elections considered.

The right plot replicates the analysis for upper chambers. In this case the VRB mostly benefits the right and the magnitude of the effect is slightly higher than in lower houses - the mean exceeds 0.02 in 9 out of 13 countries and in $69 \%$ of the elections. Notice as well that the frequently studied country cases with high degrees of malapportionment, especially Argentina, Brazil, Japan, and the United States, feature the biggest (right-wing favoring) VRB in their upper chambers. These countries may not be representative of the overall impact of legislative malapportionment, especially in nations with high degrees of lower chamber malapportionment.

Figure 5: Voter Representation Bias, by country


Note: The figure shows the mean (of the different elections) voter's representation bias, by country.

### 3.3 Relative Effects of Malapportionment in Spain: Examination of Distortion of Representation at the Individual Level

Throughout the paper we have calculated the ideology of the mean voter in a country by using the party's vote share in each district. This measure relied on three assumptions. First, voters are treated as if they have the same ideology as the party they voted for. Second, voters do not act strategically in order to maximize the utility of their vote. Third, given that we do not have the ideological position for abstainers or (often) for small parties, we assume that the ideology of these missing individuals is identical to the ideology of non-missing voters. To validate this strategy and to provide the robustness of our findings, we now turn to survey data to estimate directly the ideological position of voters and to show how it tracks voters' ideological stances derived from party vote.

Our data comes from Spain - an ideal case to study for several reasons. First, according to our data, Spain ranks fourth among all representative democracies (and first among western countries) in terms of malapportionment (see Figure (1). Second, the electoral system in Spain divides the country in 52
electoral constituencies of very different sizes - a sufficient number of districts to observe variation in each district's mean ideology. Constituency heterogeneity is exacerbated by the fact that the supply of parties as well the latter's electoral support vary considerably across constituencies. Third, for the whole period under consideration (1993-2016), the Centro de Investigaciones Sociológicas (CIS) - the national polling institute, conducted monthly surveys (except for the month of August) that included the ideology (on a scale from 1 to 10 that we have recoded to range from 0 to 10 ) and electoral district (province) of respondents. The resulting data set of all census results totals 487,942 individual observations. Attributing survey responses to the upcoming election, we end up with approximately 70,000 individual observations for each of the seven elections under consideration (1996, 2000, 2004, 2008, 2012, 2015, and 2016).

To calculate the congruence of our ideological estimates using party-level data, on the one hand, and survey-level data, on the other, we estimate the mean district ideology in the 50 Spanish multimember districts between 1996 and 2016. This strategy not only allows us to overcome the assumption that voters have the same ideology as the party they voted for, but also includes voters from small parties and abstainers for which we cannot derive their ideological stances when employing expert surveys. Figure D. 2 in the Appendix shows a high correlation between the mean district ideology using individual and party data. Figure D.2 reveals too that voters in Spain place themselves about one point to the left of the estimate obtained by using party vote share. The deviation, however, is systematic across the different values of ideology, thus not affecting the correspondence between both levels of data: the correlation is 0.71 for the lower house elections and 0.64 for upper house elections.

To identify the extent to which our measure of VRB using party vote shares is consistent with a VRB using individual-level data, we replicate the calculations in Section 3.1 with the CIS survey data. To do so, we calculate the difference between the ideology of the average voter according to population, and the average ideology after weighting each individual by her district's RRI ${ }^{11}$

Figure 6 shows the evolution of the VRB for both party data (solid line) and individual-level data (blue long dashed line) in the Chamber of Deputies (panel left) and the Senate (right panel) elections. There is a high association between the VRB-party and the VRB-individual - along with a lag-effect for

[^6]the individual-level VRB ${ }^{12}$ The coefficient of correlation for the two houses is 0.72 , and we can see that the VRB-individual is generally higher than the VRB-parties. For the Chamber of Deputies the mean VRB using individual-level data is 0.04 ( 0.03 using party data), and for the Senate is 0.16 ( 0.10 previously).

Figure 6: VRB Using Party- and Individual-level Data in Spain, 1995-2015


Note: The figures compares the Voter's Representation Bias using a party's and an individual's measure of ideology. The left panel shows the evolution of the VRB in the Chamber of Deputies. The right panel does the same for the Senate. The blue long-dashed line displays the VRB when using individual-level data to calculate the change in the mean country's ideology when malapportionment is considered and when it is not. The grey short-dashed line displays the VRB when using individual-level data to calculate the change in the median country's ideology with and without malapportionment. Finally, the continuous teal line displays the baseline measure of the VRB, using party share of votes and ideological data from the CHES to calculate changes in the mean country's ideology.

Finally, as mentioned in Section 2.2 an additional concern in our analysis is that the calculation of the VRB employs the mean value of ideology instead of the median value. To address this concern, we calculate the median VRB by using individual-level data in Spain, along with a continuous measure of ideology obtained by assigning a random component to each individual's ideology. The short-dashed line in Figure 6 shows the correspondence of this measure with the ones using the mean VRB. Details about the empirical strategy and an in-depth discussion of the results can also be found in Appendix F

[^7]
## 4 Size: The Relative Distortionary Effect of Malapportionment

While the VRB uniquely identifies the effect of malapportionment in the representation of voters (excluding other components of the electoral system), the TRB includes all the effects of electoral laws such as district magnitude, the electoral formula, or the use of representation thresholds. In addition, while the TRB includes a random component attributable to the exercise of translating votes into seats, the VRB does not. In other words, in a perfectly apportioned system, the VRB would always take a value of 0 . By contrast, even in a highly proportional electoral system, the TRB will always be different than 0 , even if the effect is due to randomness, because votes have to be translated into a smaller quantity - seats.

Calculating the share of TRB generated by VRB allows us to estimate the relative weight of malapportionment in distorting democratic representation vis-à-vis other electoral rules and institutions. We do it in two steps: first we analyze the relationship between VRB and TRB as captured by our data. Second, we turn to simulation analysis and evaluate how much overall bias changes in response to modifications of malapportionment.

### 4.1 Bias Share

We calculate the share of the TRB attributable to the VRB, which we term "BiasShare" as follows ${ }^{13}$

$$
\text { BiasShare }=|V R B| /(|T R B-V R B|+|V R B|)
$$

The hollow bars in Figure 2 display the share of the total ideological distortion that is directly attributable to the VRB - and thus, to malapportionment. In the lower house elections (left panel), BiasShare is rather small for most of the countries. The share of the deviation attributable to malapportionment is below 10 percent in 55 percent of the country-elections and from 10 to 20 percent in one sixth of all observations. It is only between 20 and 30 percent in 6 percent of the cases and in 10 percent between 30 and 50 percent. In the remaining 11 percent of the country-elections malapportionment has an impact above 50 percent. Countries with elections in which malapportionment has the biggest effect in the total distortion of representation include Ecuador, Georgia, India, Japan or Slovenia.

[^8]In upper house elections, the share of TRB attributable to malapportionment is higher than in lower chamber elections. BiasShare is above 10 percent in 74 percent of the country-election observations, above 30 percent in one fourth of all of the observations, and over 50 percent in 6 percent of all cases. The countries with the upper chamber delivering more representation bias through malapportionment are the Czech Republic, Japan, Spain, and the United States. A more detailed analysis of the relationship between the VRB and the TRB reveals that, in both lower and upper house elections, the largest shares of representation bias attributable to malapportionment take place in country-elections with relatively small degrees of total representation bias (see a twoway scatter plot between the VRB and the TRB in Figure E.1] in Appendix E. This implies that the representation distortion seen in most nations with high degrees of total representation bias is emerging from sources other than lower house malapportionment.

### 4.2 Electoral Systems, Malapportionment, and the Distortion of Representation: Simulation Analysis

So far, we have assessed separately and cross-nationally the relative importance of malapportionment on distortion in representation. By their nature, however, these comparisons do not allow us to identify how changes in malapportionment shape, jointly, the two magnitudes of interest in this piece: how does an exogenous change of malapportionment shape the direction and size of the bias in the system of representation within units? Accordingly, we turn to simulation models to estimate the weight of the different features of an electoral system in distorting representation.

### 4.2.1 Method

To address this question we take advantage of the Spanish electoral law, which establishes that all 50 multi-member constituencies should send a minimum of two representatives to the Chamber of Deputies regardless of their population ${ }^{[14}$ All subsequent seats, up to the total number of 350 deputies serving in the lower house, are allocated strictly based on the district's population. This electoral design yields a malapportionment of around 0.10 for the lower house.

We manipulate malapportionment by changing the minimum (mandatory) number of seats granted to each district (except Ceuta and Melilla) from 0 to $7{ }^{[15}$ Table 2 shows the main statistic descriptives of

[^9]the eight different seat allocation strategies using data for the 2015 election: the minimum number of seats granted to each district, the total number of seats to be allocated according to population, district magnitude (mean, median, standard deviation, and maximum), and malapportionment.

When no minimum seats are assigned to every district, the share of representatives of the district (over the total representatives in the national parliament) becomes very close to the district's share of population. For instance, when comparing the status quo (2 seats granted to each district) versus a system granting no minimum seats, 6 new constituencies now elect one seat, and 5 new constituencies elect 2 . By contrast, in the election of 2015, there were only two single-member constituencies and one two-seat constituency. The level of malapportionment of this simulation model is very low, 0.036. Increasing the number of minimum seats granted to electoral districts raises the level of malapportionment - doubling the status quo level to 0.197 when there are four required seats and up to 0.339 when districts have seven seats each.

Table 2: Simulation Analysis for Spain 2015

| Minimum seats to each district | 0 | 1 | $2^{*}$ | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total seats allocated by population | 348 | 298 | 248 | 198 | 148 | 98 | 48 | 0 |
| Mean seats per district | 6.96 | 6.96 | 6.96 | 6.96 | 6.96 | 6.96 | 6.96 | 6.96 |
| Median seats per district | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 |
| Standard deviation | 8.68 | 7.37 | 6.12 | 4.88 | 3.69 | 2.52 | 1.34 | 0.20 |
| Largest district seats | 48 | 42 | 36 | 30 | 24 | 19 | 13 | 7 |
| Malapportionment | 0.036 | 0.060 | 0.010 | 0.148 | 0.197 | 0.248 | 0.291 | 0.339 |

* Current allocation system.


### 4.2.2 The Relative Impact of Malapportionment

To estimate VRB and its size compared to TRB for each one of the scenarios we have simulated, we calculate VRB using the configuration of party votes in Spain's 2015 election and party's ideological stances coming from the Chapel Hill Experts Survey (Ryan et al. 2020). In addition, the simulation results allow us to establish the distortionary weight of the other electoral institutions: district magnitude and the 3 percent legal electoral threshold imposed by Spanish law at the district level.

To measure the role of district magnitude in triggering representation bias, we multiply the size of all districts (in each of the 8 simulation settings) by a factor of 10 to get rid of the mechanical effect of electoral laws. According to Cox (1997), districts above 5 deputies grant fully proportional results. However, and out of caution, we apply a factor of 10 to each district - hence, the smallest districts have

10 deputies (Ceuta and Melilla) and the largest one has 480 (Madrid, in simulation 0). This allows us to determine the impact of district magnitude on TRB by comparing our new results to the baseline model. Likewise, we re-estimate our previous simulations model (where district size was multiplied by a factor of 10 ), after removing the $3 \%$ electoral threshold. Again in this case, the difference between the share of the TRB that can be accounted for by malapportionment between the baseline model and the model using the factor 10 and no electoral threshold will show the share of the TRB that can be attributable to the presence of an electoral barrier ${ }^{16}$

Figure 7 plots the minimum seats assigned to each district in the horizontal axis. The left vertical axis indicates the level of the VRB (represented by blue columns) once we modify the Spanish electoral law and change the number of minimum seats per districts (and, accordingly, malapportionment). This ranges between nearly no VRB - when no seats are imposed - to 0.13 - when all constituencies have the same district magnitude. The positive values of the VRB indicate a right-wing bias of malapportionment - an unsurprising result because Spain's less populated provinces, which benefit from the 2 -seat requirement, tend to favor the right.

The right vertical axis measures the level of malapportionment, as well as BiasShare, that is, the percentage of the total representation bias that can be accounted by malapportionment. The black continuous line in Figure 7. which represents the level of malapportionment, shows the latter increases with the minimum mandatory number of seats. BiasShare is drawn under the different simulation scenarios we have run: without changing district magnitude or the electoral threshold; modifying district magnitude; and dropping the threshold requirement. BiasShare, as displayed by the short-dashed teal line, does too. When no seat is imposed to electoral constituencies, both the level of malapportionment and BiasShare are close to zero. In the status quo model (2 required seats), malapportionment represents 22.5 percent of BiasShare. This percentage progressively increases until we reach the 5 seats granted to districts, where almost half of the TRB can be accounted by malapportionment.

Table 3 complements Figure 7 by reporting the percentage of the total bias that can be accounted by malapportionment, district magnitude and the electoral barrier, across the eight different simulation models. The last column displays the percentage of the bias than cannot be explained by these institutions and, hence, should be attributed to randomness. District magnitude is the main driver of

[^10]Figure 7: The role of institutions in triggering representation bias given changing levels of malapportionment, Spain 2015


Note: The figure shows the level of VRB (left axis and columns) as we increase the level of malapportionment from a totally proportional distribution of seats to a highly malapportioned system. The straight black line shows the magnitude of malapportionment Samuels and Snyder 2001. The dashed teal line shows the percentage of the TRB that can be accounted by malapportionment; the short-dashed maroon line shows the same indicator when we eliminate the constraining effect of district magnitude from the equation; the long-dashed navy line additionally reduces the $3 \%$ district-level electoral threshold present in the Spanish electoral law.
representation bias. In the status quo simulation (a minimum of 2 seats), district magnitude accounts for half of the TRB, and even when we artificially increase malapportionment, the role of this institution in triggering bias is very high. By contrast, the electoral threshold has a small effect. Its impact ranges between $5 \%$ and $10 \%$ across models. ${ }^{17}$,

Table 3: Institutions and representation bias across different simulation models, Spain 2015

[^11]| Minimum seats | \% malapp. | \% DM | \% threshold | \% random |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $1.3 \%$ | $13.6 \%$ | $5.0 \%$ | $80.2 \%$ |
| 1 | $11.6 \%$ | $54.8 \%$ | $11.3 \%$ | $22.3 \%$ |
| 2 | $22.5 \%$ | $49.1 \%$ | $4.8 \%$ | $23.7 \%$ |
| 3 | $29.6 \%$ | $50.1 \%$ | $9.5 \%$ | $10.9 \%$ |
| 4 | $41.1 \%$ | $45.7 \%$ | $9.9 \%$ | $3.3 \%$ |
| 5 | $47.5 \%$ | $39.5 \%$ | $3.8 \%$ | $9.2 \%$ |
| 6 | $49.1 \%$ | $41.4 \%$ | $7.6 \%$ | $1.9 \%$ |
| 7 | $46.9 \%$ | $49.6 \%$ | $-0.9 \%$ | $4.5 \%$ |

* Current allocation system.


## 5 Conclusions

Political inequality is, to a considerable extent, the outcome of the broad set of institutions and norms that structure representation. In this paper we have focused on one of its least researched facets: malapportionment. With that end in mind, we have put together a novel and comprehensive data set with measures of malapportionment at the district and national levels as well as the ideological composition of voters and legislatures covering over 19,000 electoral districts in 65 democratic countries. In addition, we have engaged in a set of simulations that have relied on survey data encompassing almost 490,000 Spanish respondents. Both exercises have allowed us to estimate the size of malapportionment, the direction in which it biases policy, and its relative importance relative to other sources of political bias.

Our examination of malapportionment delivers two key findings. First, the direction of the ideological bias associated with malapportionment is heterogenous in nature - contradicting the current consensus in the literature. Malapportionment favors right-wing-leaning territories in several important cases, and in most cases of upper house malapportionment. But its effects are ideological neutral or indeed take the reverse direction, favoring left parties, in more than half of our observations in lower legislative houses. As a matter of fact, malapportionment works in opposite directions in the lower and upper houses of several countries. These results question an influential literature that has associated malapportionment to smaller welfare states.

Second, although malapportionment emerges as a relevant source of political bias among democracies, its effects do not overwhelm other institutional factors. Our calculations suggest that, on average, malapportionment accounts for about 10 to 15 percent of the total deviation of parliaments from the
preferences of voters. Nonetheless, there is substantial heterogeneity in the incidence of malapportionment as a source of bias. The share of the deviation attributable to malapportionment remains below 10 percent in over half of the country-elections and ranges from 10 to 20 percent in one sixth of all observations. Malapportionment only has an impact above 50 percent in one tenth of our observations. These cross-national results are also confirmed by a simulation analysis, done within one country, that measures voters' preferences directly and that enables us to isolate the effect of each electoral rule separately

Malapportionment is one source of ideological bias among several in democracies. Our analysis has taken as much observational data as possible to see the extent of that bias and its direction. As we push our agenda forward, we will explore several avenues to identify the role of institutions and other factors on the degree of ideological bias. First, given that observed ideological data are shaped by the current level of malapportionment and the strategic and psychological effects of existing institutions, we estimate the contributions of these factors in a fully simulated exercise. Second, we take our analysis into the legislative arena, to see whether malapportionment shapes redistributive voting once other institutions are taken into account.

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## Online Appendix

## A Baseline Data: Additional Information and Sources

## A Voter Ideology

To maximize the number of countries for which we can derive the ideology of the voters at the district level, we employ several expert surveys. In particular, we rely on the following surveys:

1. Chapel Hill Experts Survey (Ryan et al. 2020) [CHES], a longitudinal dataset on party stances in 31 countries in Europe, built from surveys to country experts, with data ranging from the late 1990s until 2019, and containing party's positions on economic ideology, along with several other measures of cultural ideology, GAL/TAN, preferences for redistribution, etc.
2. Political Representation, Executives, and Political Parties Survey (Wiesehomeier, Singer and RuthLovell 2019) [PREPPS], a one-time database on party stances based on expert surveys conducted between the fall of 2018 and the summer of 2019 in Latin American countries. PREPPS registers party's ideological stances for most of the parties in 19 Central and South American countries. Similarly to CHES, PREPPS contains a large number of parties - something that is crucial in the usually fragmented party systems in Latin America. While the strength of this data set is its party coverage, its ideology measure conflates the cultural and economic dimensions.
3. V-Dem Party Survey (Lührmann et al. 2020) [V-Dem], which includes the ideological positions for 3,489 political parties in 169 countries between 1900 and 2019. Despite the longitudinal character of the data, small parties are rarely included in the database, thus not covering, in some occasions, a considerable portion of the share of votes at the district level.
4. Global Party Survey (Norris 2020) [GPS], which results from a collaboration with CHES to expand the latter's comparative framework and policy issues outside of Europe. GPS contains one-time party positions (collected at the end of 2019) for 163 countries in the world and more than 1,000 parties. The number of parties for the 2019 elections is bigger than the one for the V-Dem but data is available only for one point in time.

The ideology indicator ranges from 0 to 10 in CHES and GPS, from -5 to 5 in V-Dem, and from 1 to 20 in PREPPS. Lower (higher) values correspond to left-(right-)wing stances. We have recoded the ideology variable in PREPPS and V-Dem on a range from 0 to 10. Table A.1 displays the exact wording
for the (economic) ideology variable in the four different data sets used in the cross-country analysis: the Chapel Hill Experts Survey (CHES) (Ryan et al. 2020), the Political Representation, Executives, and Political Parties Survey (PREPPS) (Wiesehomeier, Singer and Ruth-Lovell, 2019), the V-Dem Party Survey (V-Dem) (Lührmann et al. 2020), and the Global Party Survey (GPS) (Norris, 2020). For the CHES, the V-Dem and the GPS data sets we use a measure of economic ideology. The wording of this variable is identical for the CHES and the PREPPS data sets, and very similar in the V-Dem. Finally, the PREPPS data set does not include a measure of economic ideology and we use a measure of placement "on a general left-right dimension, taking all aspects of party policy into account".

Table A.1: Wording for the economic ideology variable in the different datasets

| Database | Wording | Categories |
| :---: | :---: | :---: |
| CHES | Parties can be classified by their current stance on ECONOMIC ISSUES such as privatization, taxes, regulation, government spending, and the welfare state. Those on the economic LEFT want government to play an active role in the economy. Those on the economic RIGHT favor a reduced role for government. Where would you place each party on the following scale? | 0: Extreme Economic Left, $\quad 10:$ Extreme Economic Right |
| PREPPS | Please locate each party on a general left-right dimension, taking all aspects of party policy into account. | Left (1), Right (20) |
| V-Dem | Please locate the party in terms of its overall ideological stance on economic issues. Clarification: Parties on the economic left want government to play an active role in the economy. This includes higher taxes, more regulation and government spending and a more generous welfare state. Parties on the economic right emphasize a reduced economic role for government: privatization, lower taxes, less regulation, less government spending, and a leaner welfare state. | -5: Far-left, 5: Far-right |
| GPS | Parties can be classified by their current stance on ECONOMIC ISSUES such as privatization, taxes, regulation, government spending, and the welfare state. Those on the economic LEFT want government to play an active role in the economy. Those on the economic RIGHT favor a reduced role for government. Where would you place each party on the following scale? | 0: Extreme Economic Left, $\quad 10:$ Extreme Economic Right |

Finally, in each election we select the source of data that allows us to maximise the share of votes cast for which we have data on ideology matched. This means that, although in each election we consistently use the same source of data - we do not code some districts with a database and others with another - , in some occasions, certain election-years may be covered with a given database and other election-years with a different one ${ }^{18}$ As a result, we cover 158 elections ( $54 \%$ of the sample) with

[^12]CHES, 19 elections (7\%) with PREPPS, 72 elections (25\%) using V-Dem, and 41 elections (14\%) with GPS ${ }^{19}$ To avoid the inclusion of elections with poor coverage, we have discarded all elections for which the mean coverage across districts is below $66 \%$ of the share of votes cast. Figure B.1 in the Appendix displays the availability of ideological data at the district level by country.

## B Malapportionment

Figure A. 1 shows the distribution of (log)RRI in our sample. The columns in blue depict the malapportionment values at the district level in all lower chamber elections in our data. The columns in red graph the distribution for upper chamber elections. District malapportionment is higher in the latter than in the former.

Figure A.1: Distribution of District-level Malapportionment in Lower and Upper Houses


Note: The figure displays the logarithmic transformation of the district-level Relative Representation Index from Ansolabehere et al. 2002) in lower (blue) and upper (red) chamber elections. A value of 1 means that the district is perfectly apportioned, values below (above) 1 represent under(over)represented districts. The note below the legend displays the mean and standard deviation (between brackets) of the distribution of malapportionment for lower and upper chamber elections.

Finally, Figure A. 2 shows the distribution of district-level malapportionment values separately for each country (lower and upper house elections plotted together for bicameral countries). District

[^13]malapportionment varies substantially across countries. While it is practically absent in Finland, South Africa, Latvia or New Zealand, it is very high in Argentina, Bolivia, Spain, Malaysia or Zambia.

Figure A.2: District-level Malapportionment by Country

(log) Relative Representation Index

Note: The figure displays the logarithmic transformation of the district-level relative representation index from Ansolabehere et al. 2002 across countries, in lower and upper house elections.

## B Descriptives

Table B.1 displays the country and election years included in the empirical analysis for lower house elections. Table B.2 does the same for upper house elections.

Table B.1: Countries and elections included in the dataset (Lower House elections)

| Albania | 2013 | 2017 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Argentina | 2011 | 2013 | 2015 |  |  |
| Australia | 2010 | 1999 |  |  |  |
| Austria | 2002 | 2006 | 2008 | 2013 |  |
| Belgium | 1999 | 2003 | 2007 | 2010 | 2014 |
| Bolivia | 2009 |  |  |  |  |

Table B. 1 Continued: Countries and elections included in the dataset Lower House elections)

| Botswana | 2014 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Brazil | 2010 | 2014 |  |  |  |  |  |
| Bulgaria | 2001 | 2005 | 2013 | 2014 |  |  |  |
| Canada | 2015 |  |  |  |  |  |  |
| Cape Verde | 2011 | 2016 |  |  |  |  |  |
| Chile | 2009 | 2013 |  |  |  |  |  |
| Colombia | 2014 |  |  |  |  |  |  |
| Comoros | 2015 |  |  |  |  |  |  |
| Costa Rica | 2010 | 2014 |  |  |  |  |  |
| Croatia | 2000 | 2003 | 2007 | 2011 | 2015 | 2016 |  |
| Cyprus | 2001 | 2006 | 2011 | 2016 |  |  |  |
| Czech Republic | 1996 | 1998 | 2002 | 2006 | 2010 | 2013 | 2017 |
| Denmark | 1998 | 2001 | 2005 | 2007 | 2011 | 2015 |  |
| Dominican Republic | 2010 | 2016 |  |  |  |  |  |
| Ecuador | 2013 |  |  |  |  |  |  |
| El Salvador | 2012 | 2015 |  |  |  |  |  |
| Estonia | 1999 | 2003 | 2007 | 2011 | 2015 |  |  |
| Finland | 1999 | 2003 | 2007 | 2011 | 2015 |  |  |
| France | 1997 | 2002 | 2007 | 2012 | 2017 |  |  |
| Georgia | 2016 |  |  |  |  |  |  |
| Germany | 1998 | 2002 | 2005 | 2009 | 2013 |  |  |
| Ghana | 2012 | 2016 |  |  |  |  |  |
| Greece | 1996 | 2000 | 2004 | 2007 | 2009 | 2012 (2) | 2015 (2) |
| Guatemala | 2015 |  |  |  |  |  |  |
| Guinea-Bissau | 2014 |  |  |  |  |  |  |
| Honduras | 2013 | 2017 |  |  |  |  |  |
| Hungary | 1998 | 2002 | 2006 | 2010 | 2014 |  |  |
| Iceland | 2016 | 2017 |  |  |  |  |  |
| India | 2014 |  |  |  |  |  |  |
| Ireland | 1997 | 2002 | 2007 | 2011 | 2016 |  |  |
| Italy | 1996 | 2001 | 2006 | 2008 | 2013 | 2018 |  |
| Japan | 2009 | 2012 | 2014 |  |  |  |  |
| Korea | 2012 |  |  |  |  |  |  |
| Latvia | 1998 |  |  |  |  |  |  |
| Latvia | 2002 | 2006 | 2010 | 2011 | 2014 |  |  |
| Lesotho | 2012 | 2015 | 2017 |  |  |  |  |
| Lithuania | 1996 | 2000 | 2004 | 2008 | 2012 |  |  |
| Macedonia | 2014 |  |  |  |  |  |  |
| Malawi | 2014 |  |  |  |  |  |  |
| Malaysia | 2018 |  |  |  |  |  |  |
| Malta | 1996 | 1998 | 2003 | 2008 | 2013 | 2017 |  |
| Mauritius | 2010 | 2014 |  |  |  |  |  |
| Mexico | 2009 | 2012 | 2018 |  |  |  |  |
| Mongolia | 2012 |  |  |  |  |  |  |
| New Zealand | 2014 |  |  |  |  |  |  |
| Norway | 2013 | 2017 |  |  |  |  |  |
| Peru | 2011 |  |  |  |  |  |  |
| Poland | 1997 | 2001 | 2005 | 2007 | 2011 | 2015 |  |
| Portugal | 1999 | 2002 | 2005 | 2009 | 2011 | 2015 |  |
| Romania | 1996 | 2000 | 2004 | 2012 | 2016 |  |  |
| Slovenia | 2000 | 2004 | 2008 | 2011 | 2014 |  |  |

Table B.1 Continued: Countries and elections included in the dataset Lower House elections)

| South Africa | 2014 |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Spain | 1996 | 2000 | 2004 | 2008 | 2011 | 2015 | 2016 |
| Sweden | 1998 | 2002 | 2006 | 2010 | 2014 |  |  |
| Switzerland | 2015 |  |  |  |  |  |  |
| Turkey | 2011 | 2015 | 2015 |  |  |  |  |
| UK | 2001 | 2005 | 2010 | 2015 | 2017 |  |  |
| Uruguay | 2014 |  |  |  |  |  |  |
| US | 2016 |  |  |  |  |  |  |
| Zambia | 2011 | 2016 |  |  |  |  |  |

Table B.2: Countries and elections included in the dataset (Upper House elections)

| Argentina | 2001 | 2003 | 2009 |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Bolivia | 2002 | 2005 | 2009 | 2014 |  |  |  |  |  |  |
| Brazil | 2002 | 2006 | 2010 | 2014 | 2018 |  |  |  |  |  |
| Chile | 2001 | 2005 | 2009 | 2013 |  |  |  |  |  |  |
| Czech Republic | 2000 | 2002 | 2004 | 2006 | 2008 | 2010 | 2012 | 2014 | 2016 |  |
| Dominican Republic | 2020 |  |  |  |  |  |  |  |  |  |
| Japan | 2001 | 2004 | 2007 | 2010 | 2013 |  |  |  |  |  |
| Liberia | 2020 |  |  |  |  |  |  |  |  |  |
| Mexico | 2000 | 2006 | 2012 |  |  |  |  |  |  |  |
| Poland | 2019 |  |  |  |  |  |  |  |  |  |
| Romania | 2008 | 2012 |  |  |  |  |  |  |  |  |
| Spain | 2000 | 2004 | 2008 | 2011 | 2015 |  |  |  |  |  |
| US | 2000 | 2002 | 2004 | 2006 | 2008 | 2010 | 2012 | 2014 | 2016 |  |

Figure B. 1 displays a series of histograms, by country, showing the availability of ideological data at the district level in each election. For each election we use the data set that renders a higher coverage. A coverage equal to 1 means that, for all the elections included in the analysis, we have complete data for all parties and all districts. In other words, that we have the ideology of all parties competing in these elections. Values close to 1 reveal that we have ideological data for the larger parties in the country but we may lack it for minor parties - often parties with no representation. The vertical line at the value 0.66 represents the election threshold of inclusion. When the mean coverage across districts in an election is lower than 0.66 , we do not include this election in the analysis. We prefer to remove entire elections instead of selected districts in order to prevent incurring in a selection bias. Changing the election threshold of inclusion does not substantively change the results, but only the number of elections that are included in the analysis.

Figure B.1: Availability of ideological data at the district level (\% votes), by country


Note: The figure displays a series of histograms, by country, of the share of parties (according to the percentage of vote) for which there is availability of ideological data. The 0.66 line displays the cut-off point beyond which we include an election in the analysis.

Figure B. 2 displays the relationship between the level of malapportionment - measured through the (log) Relative Representation Index - and each district's share of population entitled to vote as compare to the total population entitled to vote in a given election, by country. A negative coefficient indicates that, as the district's share of population increases, the district becomes more underrepresented (and vice versa). The evidence is clear in showing that in most countries of the world larger districts in terms of population are granted a smaller proportion of seats to what could be expected, yet there are notorious exceptions. In Australia, Belgium, Bulgaria, Guatemala and Portugal, if any, larger districts are overrepresented.

Figure B.2: Are small districts more malapportioned?


Note: The figure displays the relationship between the level of malapportionment - measured through the (log) Relative Representation Index - and each district's share of population entitled to vote as compare to the total population entitled to vote in a given election, by country.

## C Measuring ideology

To assess the reliability, validity and comparability of these surveys, Figure C.1 displays scatter plots of the pairwise comparisons of the mean district economic ideology between the four surveys, along with the correlation and the number of observations included in each comparison. Each dot indicates a district's mean ideology.

The figures reveal a very high consistency across data sets in assessing party's (economic) ideology. The coefficient of correlation is above 0.85 for the CHES-VDEM and the CHES-GPS comparisons, and over 0.76 for the VDEM-GPD and VDEM-PREPPS comparisons - CHES and PREPPS cannot be compared because there is no overlap between the two data sets.

Figure C.1: Scatterplot between the different sources of ideology


Note: The figure displays pairwise comparisons of the mean district economic ideology between the different sources of data. The CHES-PREPPS comparison is not available due to the lack of overlap between the two databases. The bottom-right corner of each panel displays the correlation and the number of observations (districts) for which the comparison is done. Finally, the diagonal line reveals where values would be placed if the mean ideology at the district level would be the same in the two data sets.

## D Malapportionment and ideology

Figure D.1 displays a series of scatterplots of the (log) Relative Representation Index and the mean economic ideology in the district. A positive (negative) value reveals that malapportionment is higher in rightist (leftist) districts. The evidence is completely mixed and, far from conventional wisdom, it is not possible to assert that malapportionment systematically benefits the right/right-wing parties.

Figure D.1: Are more malapportioned districts more economically conservative?


Note: The figure displays a series of scatterplots of the (log) Relative Representation Index and the mean economic ideology in the district.

Figure D.2 shows the correlation between the district's mean ideology using individual- and party-level data coming from the CHES in Lower (blue circles) and Upper (red diamond) house elections in Spain between 1996 and 2015. The diagonal line displays an equivalence line for the individual- and party-level ideology at the district level. The fact that the mean ideology using individual-level data is below this diagonal line reveals that, in Spain, people place themselves more to the left than (the CHES evaluation of) the parties they vote for.

Figure D.2: Correlation between the mean district ideology using individual and party level data in Lower and Upper house elections in Spain, 1996-2015


Note: This figure shows the correlation between the district's mean ideology using individual- and party-level data in Lower (blue circles) and Upper (red diamond) house elections in Spain between 1996 and 2015. The diagonal line displays an equivalence line for the individual- and party-level ideology at the district level.

## E Malapportionment and the total representation bias

Figure E. 1 plots the BiasShare ( y -axis) against the TRB (x-axis). Particularly in lower house elections (blue dots), the largest shares of representation bias attributable to malapportionment take place in country-elections with relatively small degrees of overall representation bias. In upper chambers (red dots) the trend is very similar yet some relatively high shares of bias attributable to malapportionment can be found in country-elections with small-to-medium total representation bias.

Figure E.1: Share of bias attributable to malapportionmment and the total representation bias


Note: The figure scatter plots the Total Representation Bias against the share of the total representation bias that can be directly attributable to malapportionment (BiasShare) in lower and upper chambers by country. The size of the markers is proportional to the magnitude of malapportionment (Samuels and Snyder 2001).

## F The median distortion of representation

As explained in the body of the manuscript, an important concern is that the calculation of the VRB employs the mean district ideology before and after malapportionment is taken into consideration. In section 2.3 we have explained why, for our purpose, it is better to use the mean value to calculate the VRB instead of the median value. In a nutshell, if ideology can take a limited number of values - for the party analysis, the ideology position of each party; for the survey analysis, the 10 categories of the ideology variable - then observing a change in the median voter mostly depends on the distribution of the variables of interest. In this case, changes in the median ideology due to malapportionment will occur when the districts in a country are highly malapportioned and the distribution of party support / voters is structured in a way that around the median voter there is a change in ideology.

Figure F. 1 displays the median (of the different elections) voter's representation bias, by country. The figure confirms that, when using the median ideology, the VRB rarely changes in lower house elections, while this is relatively more frequent in upper chambers - due to higher levels of malapportionment.

In order to assess the relationship between the median and the mean VRB, Figure F.2 displays a scatter plot between the two measures of the VRB. The figure shows that only when the VRB-mean is high and thus, malapportionment is also high -, the VRB-median changes. However, most of the changes in the VRB that fall within the -0.05 and +0.05 range show no change in the median VRB.

When using individual-level data, this situation is aggravated by the fact that ideology is often normally distributed and, hence, most individuals are concentrated around the median (values 4 to 6 ) - thus making it even more difficult to observe variation due to malapportionment (but also due to other electoral institutions). In fact, using individual-level data from the CIS in Spain as in Section 3.3, the calculation of the median VRB does not generate any substantive deviation in the VRB-individual once we include malapportionment. This is because, despite the distribution of the ideology variable changes from one election to another, the median value always correspond to the value 4 - which, as it happens, it is also always the mode.

One way to tackle this is to transform the 10-categories ideology variable in the survey into a continuous measure by introducing some random noise component into each respondents' ideological selfplacement. To do so, we create a new random and uniformly distributed variable for each individual that ranges from -0.5 to 0.5 . Next, we sum up the original individual's ideology variable in the CIS

Figure F.1: Voter's Representation Bias using the median ideology, by country


Note: The figure shows the median (of the different elections) voter's representation bias, by country. Empty bars show the (country-election mean) of the level of malapportionment in each country. Positive values reveal higher degrees of malapportionment.
survey by this random component. We end with a variable that ranges from 0 to 10 and has the same distribution than the original one, yet it contains a continuous measure of ideology. Hence, for an individual that was placed in the value 3 of the scale, the introduction of this disturbance term will randomly shift her ideology to range from 2.5 to 3.5 . The correlation between the two measures is close to $99 \%$, and figure F.3 shows the distribution of the two variables - blue columns display the original categorical distribution of the variable, teal columns show how the variable changes when we introduce the random component to make it continuous.

The results for the VRB using the mean and the median VRB can be found in Figure 6, The solid line shows the evolution of the mean VRB using party data, and the red short dashed line in displays the evolution of the VRB in lower and upper house elections in Spain between 1996 and 2015 when using the VRB median. The figure reveals the existence of notorious parallel trends between this variable and the VRB measure using party data ( $\mathrm{r}=0.73$ ), and particularly between the VRB mean and median

Figure F.2: Association between the VRB mean and VRB median


Note: The figure plots the value of the Voters' Representation Bias when using the mean value of the deviation (x-axis) against the Voters' Representation Bias when using the median value of the deviation ( y -axis). The horizontal line cutting across the 0 value signals all the observations for which the VRB does not change when considering how malapportionment displaces the median voter. The vertical line does the same for the mean voter. The size of the markers is proportional to the magnitude of malapportionment (2001).
measures using individual-level data ( $\mathrm{r}=0.98$ ).

As for the size of the VRB measure using the median value, this is $70 \%$ as big as the one using individuallevel data and the mean value; finally, the VRB measure using party-level data is $90 \%$ as big as the VRB measure using individual-level data and the median value. Overall, this evidence reveals the consistency of our measures using different empirical strategies and is a proof of the robustness of our findings using observational data.

Figure F.3: Histogram of ideology in Spain, 1995-2015


Note: Blue columns in the figure display the histogram of ideology using original data from the CIS ( $\mathrm{N}=476,631$ ). Red columns display the distorted measure of ideology, where each ideological category has modified by adding into it a random term that ranges from -0.5 to 0.5 . The correlation between the two measures is $99 \%$.


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[^1]:    ${ }^{2}$ We follow Samuels and Snyder 2001 to calculate legislative malapportionment at the country level. This is obtained by taking the absolute value of the difference between each district $i$ 's seat ( $s_{i}$ ) and population shares ( $\nu_{i}$ ), adding them, and then dividing it by two (Samuels and Snyder 2001):

    $$
    M A L_{n a t}=(1 / 2) \sum\left|s_{i}-v_{i}\right|
    $$

    ${ }^{3}$ Exceptionally, Czech Republic and Romania have low levels of malapportionment in both lower and upper house elections.

[^2]:    ${ }^{4}$ Other research has also reported the pro-elite bias of malapportionment, despite not necessarily related to an ideology. Boone and Wahman 2015, using data from eight African countries with SMD, show that the existence of a rural-bias in apportionment accentuates the incumbent bias in parliamentary representation. Rodden (2002) reveals the existence of a positive bias of overrepresented member states in the distribution of fiscal transfers in the European Union. Bhavnani 2018) finds that legislative malapportionment translates to cabinet malapportionment in parlimentary democracies such as India.

[^3]:    ${ }^{5} \mathrm{~A}$ particular case of concern is the allocation of seats granted to minorities. In some occasions this is done in a large district where seats are also allocated to non-minority parties. We have dropped seats allocated to ethnic minorities in non-specific minority constituencies only when the sum of seats distributed to parties is different from the declared district's magnitude.
    ${ }^{6}$ Further details on the sources and procedures employed in these calculations are available in Appendix A The list of countries and elections can be found in the Appendix in Table B.1 for lower house elections and Table B.2 for upper house elections.

[^4]:    ${ }^{7}$ To determine the district-level degree of malapportionment we could simply rely on the (absolute) difference between the share of votes and the share of seats at the district level - this is, the first step in the Samuel and Snyder's 2001 measure ${ }^{8}$ However, because this measure calculates the absolute difference between two values, it is insensitive to the relative change between two values.
    ${ }^{9}$ Figure A. 2 in Appendix B displays the distribution of the (log)RRI by country.

[^5]:    ${ }^{10}$ In empirical terms, to calculate the mean voter in the country (irrespective of malapportionment) we weight each vote's ideology at the district level by the share of votes of this party in the district and the share of population of this district over the total country's population. In turn, to calculate the mean voter in the country factoring in the effect of malapportionment we weight the party's ideology by the share of votes each party has obtained in the district and the share of seats each district elects over the total number of seats in the country.

[^6]:    ${ }^{11}$ Electoral constituencies in Spain match the administrative provinces for the lower house elections. In the Senate, two territories from the Balearic Islands and five from the Canary Islands have their own constituency in which they elect a single seat; in parallel, each of the main islands from the Balearic and the Canary Islands (Mallorca, Gran Canaria, and Tenerife) elect three representatives. Given that the CIS survey does not allow us to distinguish whether respondents belong to the main island or one of the small islands that in the Senate allocate one seat, we have aggregated the electoral results to match the administrative provinces (and, thus, the data from the CIS survey). Also, given that in the CIS survey the number of observations for the African enclaves of Ceuta and Melilla is very low, we have dropped these observations from the analysis. Hence, the number of districts for the senate is 50, the same as for the analysisi of the Chamber of Deputies.

[^7]:    ${ }^{12}$ Such lag effect can be attributed to the fact that we have assigned survey responses recorded one year after a national election to the upcoming election, which usually takes place 3 years later. Alternatively, we could have assigned this response to the previous election. In any case, results are robust to the different strategies we use to match survey data to election years.

[^8]:    ${ }^{13}$ To clarify the calculation of the BiasShare, consider the following examples. In a country where VRB equals 0.1 and TRB is 0.2 , the share of representation bias attributable to malapportionment is 0.5 . By contrast, in a scenario with VRB equal to -0.1 and TRB equal to 0.1 , the bias share is 0.3 : here malapportionment moves representation to the left (from 0 to -0.1 ), yet the electoral system shifts representation to the right (to a positive value of 0.1 ). Hence, the expected positive impact of the electoral system to offset the negative impact of the VRB is +0.2 .

[^9]:    ${ }^{14}$ The Spanish lower house also has two single-member constituencies, Ceuta and Melilla.
    ${ }^{15}$ Assigning 7 seats each in 50 constituencies, plus the two deputies for Ceuta and Melilla, sum up to 352 deputies ( 2 more seats that in the original parliament). To preserve the size of the parliament, we chose to allocate just 6 seats to the two smallest multi-member districts in Spain: Soria and Teruel. Results are virtually identical if we allocate 7 seats across the board and allow the Chamber of Deputies to have 352 deputies.

[^10]:    ${ }^{16} \mathrm{We}$ have also experimented with a different allocation formula, the Saint-Laguë method, which is said to be the most proportional one Benoit 2000. However, given that district magnitude is multiplied by 10, the constraining effect of the d'Hondt method may actually provide less biased results when compared to the the Saint-Laguë method. This can be explained by the tendency of this later method to overrepresent small parties. Results using the Saint-Laguë method can be consulted in the replication materials. Finally, while ballot structure (closed vs. open list) may also contribute to bias, the closed list ballot system in Spain does not allow us to run simulations varying the structure.

[^11]:    ${ }^{17}$ In the last model ( 7 seats granted) eliminating the electoral barrier has even a negative impact on triggering representation bias given that this electoral institution in the DM factor 10 simulation was preventing a right-wing party to enter. When the barrier is released, the party enters and minimally moves the representation bias to the left.

[^12]:    ${ }^{18}$ The use of a ranking criteria for the different databases instead of the maximisation of the coverage provides similar results, despite several elections being discarded due to insufficient coverage. As an alternative strategy, we could have also chosen to average the party positions in the different databases in a view to maximize the number of observations. This strategy, nonetheless, would come at a cost: given the asymmetry of party coverage, the sources of data for different parties in the same election may be different and thus introduce some bias. In any case, our results do not vary depending on the selected criteria.

[^13]:    ${ }^{19}$ CHES is the source of ideological data for all elections taking place in any of the 31 countries in Europe that this expert survey covers. We have ideology matched with party results starting in 1996 until the present. The VDem dataset contains also longitudinal data going back (for some countries) to 1900 . We have used the CHES temporally bound (year 1996) as the first potentially covered year given that coverage becomes smaller as we go back in time. In the case of PREPPS and GPS, which were conducted in 2019, we allow ideological data to match party results in elections taken place not before 2009 (i.e., 10 years). When elections take place in a year for which there is no party coverage, we have taken the closest value in time (giving priority to early data in case of a draw). Finally, when faced with pre-electoral coalitions, we have estimated the coalition's ideological position at time $t$ by calculating the weighted arithmetic mean given electoral results at $t-1$.

