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Repression of enslaved Americans' protest: A model of escape in the antebellum South[*]

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Abstract

How did Southern elites maintain a system that violently extracted labor out of unwilling participants? Resistance by enslaved Americans was a natural byproduct of the economic system and at the same time threatened elites' wealth, power, and lives. So, Southerners employed a litany of individual and collective strategies to reduce the threat of resistance. I study how the South repressed one particular type of resistance: escape. While existing work has considered various repressive strategies in isolation, I model two ways to discourage escape - *ex ante* positive incentives and *ex post* pursuit - and contextualize them within the broader repressive environment. Preliminary results indicate that enslavers could reduce escape attempts by offering higher rewards, but under sufficiently high free-market conditions, enslaved persons receiving the highest rewards would nonetheless escape. The model offers insight into enslavers' political demands and the determinants of escape attempts.

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1 Introduction

The antebellum South does not exist without sufficient repression of enslaved Americans. While mass uprisings were rare, enslaved Americans frequently resisted in other ways. They broke tools, feigned ill, set fires, hoarded weapons, killed patrollers, killed enslavers, and ran away (Bauer and Bauer 1942; Aptheker 1943; Kilson 1964; Stampp 1956; Franklin and Schweninger 1999). And White Southerners obsessed over this resistance; their obsession itself a symptom of racist paranoia and a reflection of the rational assessment that enslaved Americans could destroy their wealth and lives.[1] White women confessed to one another that they lived in constant terror; both men and women admitted to sleeping with one or two pistols under their pillows (Aptheker 1943, 49–52). Accordingly, Whites' security concerns infiltrated every policy area, including seemingly tangential ones like wartime conscription, territorial expansion, White suffrage, and perhaps even public school funding (Keyssar 2000; Baptist 2014; Lawrimore 2023).

Enslavers' overwhelming fear, however, was not "general rebellion" but "the flight of individual slaves" (Fogel and Engerman 1974b, 243). Indeed, successful escape represented the loss of hundreds of dollars for the enslaver. Temporary flights cost days of lost labor, plus the resources required to recapture the runaway. The spectre of uncontrollable fugitives constituted an ever-present fear among enslavers and non-slaveholding Whites alike. Runaways also threatened the myths that Black Americans were happy to be enslaved and that the institution was sustainable and secure. To avoid economic calamity and quell the existential threat, enslavers sought plantation management strategies that would stem escape attempts.

A cottage industry on best plantation practices offered advice like, "a violent and passionate threat, will often scare the best disposed negro to the woods" (Affleck 1847) and "[divide] number of hands...as near equally...as possible of males and females... in order that each man may have his own wife on the premises. They then have no excuse for leaving home" (Alabama Planter 1852).[2]

1. For the value of holding wealth in the form of enslaved people, see González et al. (2017).

2. George Washington warned his overseer that constant attention was the only way to "prevent thieving and other disorders, the consequences of opportunities" (Affleck 1847).

But these tactics would have been futile without the state enforcing enslavers' property rights beyond the plantation. To ensure the recapture of a runaway, enslavers needed the White public to accept the collective responsibility for off-plantation repression. So, strategies for repression on- and off-plantations evolved in tandem until the end of the Civil War. Three types of strategies served to curb escapes: force and bribes on the plantation, public police and security, and ex post pursuit of runaways.[3]

Fogel and Engerman (1974a) offer a persuasive theory of plantation management: enslavers employed force (e.g., exertion under gang labor or violent punishment) until the marginal cost of force equaled the marginal cost of pecuniary payments, at which point enslavers could incentivize further effort by offering financial benefits. This theory, while essential to understanding internal economics of plantations, holds fixed enslavers' pursuit strategies and government policies, as well as other features of the environment, like economic opportunities for Black fugitives and the price of plantation agricultural outputs. A framework that accounts for all repressive strategies illustrates that the amount of force or reward was not only a function of the laborer's output, but also of the rest of the repressive environment. I construct a model to investigate how the features of the plantation's larger environment affected the equilibrium interaction between enslavers and enslaved on the plantation. Specifically, I focus on the enslaver's use of positive incentives vs. pursuit to discourage escape attempts. Enslaved laborer's protest decision was not only a response to their dissatisfaction on the plantation, but also to their available outside option and to the level of repression beyond the plantation. Their responsiveness to these outside factors, however, was mediated by enslavers' adjustments of plantation conditions.

To capture these dynamics, I solve a model of incomplete information in which the enslaver decides what benefits to provide on the plantation and whether or not to pursue a runaway. The enslaver is aware of, but cannot control, environmental characteristics like patrols and access to

3. This characterization is similar to that of Yanochik et al. (2001), who distinguish between internal security – preventing an enslaved laborer from escaping the plantation – and external security – apprehending and returning runaways, as well as between public external security – slave codes and patrols – and private external security – bounty hunters and runaway ads.

escape routes. The enslaved laborer, anticipating the payment they will receive in exchange for their productivity and the enslaver's investment into their pursuit should they run, chooses how much to produce and whether or not to run. I differentiate the skill of enslaved laborers in order to capture two empirical realities: 1) some enslaved Americans never ran, while others in the same conditions did, and 2) highly or specially skilled enslaved laborers faced unique challenges to and opportunities from escaping, compared to an inconspicuously-skilled laborer. Because the enslaver deduces a laborer's skills by observing the laborer's actions, I allow for uncertainty in the enslaver's assessment of skill.

I first situate my model within the literatures on state repression and coercive labor contracts. I then, in section 3, provide historical background on repression in the antebellum economy, and in section 4 I discuss enslaved Americans' economic considerations. In section 5 I set up the model, and I present the solution in section 6. In section 7, I introduce an extension of the model in which I endogenize the reward premium. I conclude with the implications for enslavers' political demands; namely, they demanded a state that could actively contribute to the effective repression of enslaved Americans, and thereby improve their security of property.

2 Literature

Enslavers confronted a problem many autocrats face: how to repress the masses without inciting backlash. Numerous political scientists have studied the range of strategies autocrats use to suppress dissent and the conditions under which repression backfires (Davenport 2007). In an analysis of the dynamic relationship between protest and repression, Shadmehr (2014) determines that "higher grievances...are more likely to incite repression" because those grievances "require more concessions" from the state. This conclusion is consistent with Acemoglu and Robinson (2000), who find that autocrats in societies with greater inequality are more likely to repress than to transition to democracy. Certain repressive approaches may be more likely to achieve the regime's goals. De Jaegher and Hoyer (2019), for example, finds that preemptive repression can be effective in deterring protest when the benefits of the protest are large, but that the repression accelerates protest when the benefits are small. Targeted repression can be effective in undermining cross-group collective action (Rozenas 2020), but it may also inadvertently hone the targets' resistance skills (Finkel 2015).[4]

Enslavers, like autocrats, were aware that too much repression can lead to backlash. But enslavers' calculus was even more complicated than that of most autocrats, because enslavers sought not just to quash dissatisfaction with the slavocracy but to extract laborers' maximum productivity.

Many have considered the puzzle of coercive labor, or how elites optimally extract effort from unwilling laborers, and they have studied the profitability of negative and positive incentives.[5] Responding to Fogel and Engerman (1974b), Tomaske and Canarella (1975) argue that enslavers preferred force-intensive, rather than bribe-intensive, contracts, while Findlay (1975) model how slaves could save the payments in order to purchase their freedom. Acemoglu and Wolitzky (2011)'s principal-agent model of coerced labor demonstrates that coercion increases effort but that "better outside options for workers reduce coercion." Chwe (1990) similarly finds that laborers with poor outside options are more likely to be victims of violence. Dari-Mattiacci (2013), in contrast, argues that positive incentives are essential to slave labor contracts, especially in work environments characterized by asymmetric information.

I build on the existing models of coercive labor but contextualize it within the external repressive environment. The enslaver's right to pursue, the public's participation in recapture, and the government's enforcement are essential features of the antebellum economy. On-plantation coercion is a limited understanding of repression in the antebellum south.

4. For more on the dynamic relationship between repression and protest, see Pierskalla (2010), Bueno de Mesquita and Shadmehr (forthcoming), and Shadmehr and Boleslavsky (2022).

5. For the the effects of government policies in more modern, but still coercive, economies, see Naidu (2010) and Naidu and Yuchtman (2013).

3 Repression in the antebellum South

> Among the many problems overseers dealt with...one of the most vexing and troublesome was runaway slaves. It was a problem that confronted the vast majority of managers, and one that seemed to have no solution. It did not seem to matter whether they cajoled, chastised, or severely punished offenders... Indeed, there seemed to be no end to the discussions about how to manage slaves, what incentives to offer, what liberties to grant, what penalties to inflict, and how to respond to slaves who refused to obey the rules (Franklin and Schweninger 1999, 236, 241)

Plantation management

The compliance and effort of enslaved Americans determined the economic success of the plantation, and thus “no aspect of slave management was considered too trivial to be omitted from consideration or debate" (Fogel and Engerman 1974b, 202). Enslavers read articles on the “management of Negroes” in *Farmers’ Register*, *Southern Cultivator*, *Southern Agriculturalist*, and *De Bow’s Review*. The historical record indicates that enslavers combined violence and bribes to maximize the daily productivity of enslaved laborers. A standard piece of advice for enslavers was to “always keep your word...in punishments as well as in rewards" (Affleck 1847). Enslavers logged laborers’ daily output during the harvest and doled out rewards or punishments accordingly (Prudhomme 1852; Affleck 1847; Fogel and Engerman 1974b). During planting and cultivation seasons, gang labor systems enforced an “assembly-line type of pressure” (Fogel and Engerman 1974b, 204). Some enslaved laborers might receive weekly pecuniary rewards, as in North Carolina in the late 1850s, when it was considered common practice to reward compliant behavior with 25 cents at the end of each week (Aptheker 1943, 64). In the long term, the most prized enslaved laborers might gain space in which to cultivate a personal garden, advance into choice positions, be hired out for short-term projects, and in rare instances, have the chance to purchase freedom (Aptheker 1943; Stampp 1956; Fogel and Engerman 1974b; Genovese 1974).

Ex post repression

Despite enslavers' efforts, escape attempts were frequent and irked enslavers routinely. So, enslavers often invested personal resources to recapture fugitives.

Manhunt

When the overseer or enslaver noticed a disappearance, he might ask his neighbors to be on alert or search the surrounding woodlands himself. He might also do nothing, because most runaways "were at large for only a short time - a few days or, at most, a few weeks - before they were caught or decided to return voluntarily" (Stampp 1956, 115). If the runaway had not returned after a few days or weeks, an enslaver might hire a slave catcher, instruct his overseer to search, or take out his dog to chase (and possibly mutilate) the runaway (Genovese 1974, 652).

Runaway slave ads

Enslavers could also purchase runaway slave ads and offer rewards for the runaway's return. Ads described the physical characteristics of the runaway, told when and where she escaped, and gave the address for where she could be returned and the reward collected. Ads were most often for men under 30 (Stampp 1956), suggesting that enslavers may have invested more in recapturing the most valuable enslaved laborers.

Enslavers sent the ad information to one or more papers in Southern cities and purchased ad space for consecutive days and weeks. Taking out an ad cost a few dollars per week, plus, contacting the paper could be time-consuming or costly.[6] The ad purchaser could go to the city himself, send the ad copy with someone else, or put it in the mail.

Franklin and Schweninger (1999, 282) claim that the rewards offered in the ads, small relative to slave cost, "revealed how confident masters were about the return of their human property." Work by Lennon (2016) supports the notion that ad rewards were in part a function of external

6. For a New Orleans newspaper, an ad posted for seven days would cost a minimum of $4, with a $1 flat fee and $0.50 for each subsequent day.

repressive institutions. He finds that ad reward amounts decreased in the Upper South following the passage of the Fugitive Slave Act, which he argues is because the act strengthened enslavers' property rights.

Punishment

Few runaways succeeded in their attempts, and the remainder faced punishments when they returned to the plantation (Stampp 1956; Franklin and Schweninger 1999). Although punishment was seen as necessary to deter other slaves who might consider a similar rebellion, excessive force was thought to be unwise and even detrimental. As with other slave management topics, writings on the appropriate dose of punishment were extensive (Affleck 1847; Aptheker 1943; Fogel and Engerman 1974b). Often, the overseers responsible for inflicting violence "were given instructions about the amount of corporal punishment to be administered, but they were also permitted a degree of latitude" (Franklin and Schweninger 1999, 239).

Public institutions

In addition to enslavers' private and ex-post investments in ads, legal and social institutions helped to ensure slaves' recapture.

Ad hoc apprehension

Local laws encouraged or mandated that White citizens apprehend any Black person they encountered. Any White citizen could confront a runaway and return her or deposit her in a nearby jail, where the runaway would be held until the enslaver collected her or she was resold (Olmsted 1860; Stampp 1956). This could be a profitable endeavor, especially for less wealthy Whites. In Georgia, "persons apprehending fugitives received a reward of two pence for every mile the slave was distant from his place of abode" (Flanders 1933). And in 1840 Arkansas, a White person who delivered a fugitive to the justice of the peace "would receive reimbursement. . .for the mileage he had traveled and was eligible for a fifteen-dollar reward" (Franklin and Schweninger 1999, 151).

Overall, this decentralized system of policing benefited enslavers by reducing lost labor time and other costs to recapture, but it relied on serendipity and cooperation from other Whites.

Patrols

Slave patrols, first originating in South Carolina in 1704, facilitated a more systematic approach to the repression of enslaved persons (Hadden 2001). Patrols - either staffed by volunteers or mandated by draft - policed town and countryside for runaway slaves (Franklin 1956). Groups of three to six men policed their assigned districts nightly and were responsible for apprehending Black passersby, investigating sleeping quarters, disrupting gatherings, and confiscating weapons. Enslavers advocated for and relied on patrols to control the runaway population, and states and localities continuously updated patrol policies through the late antebellum period (Franklin and Schweninger 1999).

Laws governing free Black Americans

A common strategy to preempt unrest was passing laws to restrict Blacks' economic activity and movement (Aptheker 1943, 70). After the Nat Turner rebellion, for instance, Maryland forbade the employment of Black migrants and fined those who remained in the state for more than 10 days at a rate of $50 per week (313). Some polities in the late antebellum period went so far as to expel or re-enslave free Blacks. Arkansas in 1859 directed its police "to order the state's handful of free Negroes to leave," and to hire out or sell those who remained after a year (Stampp 1956, 216). Around the same time, Virginia and Florida re-enslaved free Blacks convicted of certain offenses (Stampp 1956). Fogel and Engerman (1974b, 243) argue that these laws and others like them "severely reduced the value of freedom to [Blacks]." Likewise, enslaved Americans might have found it relatively more profitable to escape when they could do so to cities without such restrictions and with sizable free Black populations.

4 Enslaved Americans' economic calculus

Enslaved Americans had, by design, few options for improving their personal circumstances. They could ingratiate themselves to the enslaver in order to gain a modicum of material comfort. If they were lucky, they might be able to enjoy privileged positions later in life (e.g., a cook, nanny, artisan, or chauffeur) instead of continuing to labor under the sun in their old age. The most talented enslaved artisans or chefs might never have to work the fields.

Alternatively, they could attempt to escape to free territories in the North or in Mexico or to embed themselves with free and enslaved Black laborers in Southern or mid-Atlantic cities (Aptheker 1943; Stampp 1956; Hummel and Weingast 2006). Precarious but potentially lucrative opportunities were available for Black workers in Southern and Northern cities. And those who were most valued on the plantation might also be the most likely to succeed in the free market. Genovese (1974, 648) claims that "at least one-third of the runaways belonged to the ranks of the skilled and privilege slaves." I argue that part of the decision to escape included an assessment of their economic opportunities in the free market. A runaway's expected economic success was a function of her individual characteristics and exogenous trends that determined the wages available to any Black American. In addition to variation in legal restrictions, broader economic and demographic shifts could also change the market for free black labor. The influx of European immigrants in the 19th century, for example, resulted in a declining opportunities for black workers (Genovese 1974).

5 Model set-up

The game occurs in two time periods and consists of two players, an enslaver M and a representative enslaved person S. At the beginning of the game, Nature determines the type $\theta \in \{h, l\}$ of enslaved person S. The slave is of high type $\theta = h$ with probability ρ. The type determines crop weight c_S she is capable of producing and her prospective outside wage $w_\theta \in \mathbb{R}_{++}$. The outside wage schedule is public information. The costs and punishments of escape – the cost to the enslaver of recapture k and the punishment associated with a failed escape r – are also publicly known, as well as the premium γ for high productivity on the plantation. Utility is an additive function of the payoffs in both periods. Equation 1 specifies the enslaver's utility function, and equation 2 specifies the enslaved laborer's utility function.

The enslaved person observes the reward premium $\gamma(c_S)$ for producing output c_S. We will define $\gamma(c_S)$ as follows, where $\gamma \in (0, c_h)$.

$$\gamma(c_S) = \begin{cases} \gamma & \text{, if } c_S \geq c_h \\ 0 & \text{, if } c_S \in [c_l, c_h), \end{cases}$$

We will normalize low output c_l to 0, so that the reward γ and high output c_h capture the surplus of producing and receiving additional output for the enslaved laborer and enslaver, respectively. A high type can produce a high or low output, $c_S \in \{c_l, c_h\}$, and she chooses high output with probability ψ. If a high type produces a weight $c_S < c_h$ ($\psi = 0$), we will say that she has "concealed her type." Conversely, if a high type produces $c_S = c_h$, we will say that she has "revealed." While a high type can conceal her skill by producing a lower crop weight, a low type cannot masquerade as a high-producing type, because it is prohibitively costly for her to produce a high crop weight. A low type produces only $c_S = c_l$. At the end of time 1, the enslaver receives the profits for weight c_S, and the enslaved person receives the reward $\gamma(c_S)$.

In the second period, slaves of all types choose to run e_S with probability $\pi_\theta(c_S)$, which, for high types, is a function of their production c_S in the first time period. If the enslaved laborer stays, $e_S = 0$, she decides how much to produce c_S and receives the second period benefit $\gamma(c_S)$. High types, if they stay, always produce high in the second period, regardless of their first-period production decision. Alternatively, she runs, $e_S = 1$, and attempts to gain the outside wage w_θ. If a high type concealed on the plantation, she will still receive w_h in the free market. If she concealed and is recaptured, she will reveal her type upon return and produce the high amount c_h. Recaptured runaways of all types suffer punishment r.[7]

Having observed the production c_S in time $t = 1$ and whether or not slave S ran in time $t = 2$, enslaver M updates his belief $\mu(c_S, e_S)$ that slave S is of type $\theta = h$ according to Bayes' Rule. If the slave runs, the enslaver chooses to pursue her with probability $\sigma(c_S)$, where his decision to pursue is a function of the output c_S he observed in time $t = 1$. Pursuit requires the enslaver to expend resources k and results in recapture with probability $(1-\underline{q}) \in [0,1)$. If the enslaver does not pursue, the slave will escape successfully with some higher probability $\bar{q} \in (0,1]$, $\bar{q} > \underline{q}$. Note that enslaved laborers of all types face the same probability of successful escape, conditional on the enslaver's decision to pursue. Payoffs, including from recapture or successful escape, are realized. Expected payoffs are shown in equations 1 and 2.

Solution concept

I solve for perfect Bayesian equilibria. Each equilibrium consists of a mapping from the slave's type θ to how much the enslaved laborer produces and whether or not the laborer escapes, and a mapping from the enslaved person's behavior (c_S, e_S) to the enslaver's probability of pursuit.

Enslaver M determines his pursuit strategy $\sigma(c_S)$, which is the probability with which he pursues someone who produced c_S and ran, based on his belief $\mu(c_S, e_S)$ that the laborer is of type $\theta = h$. Pursuit costs the enslaver some amount of resources k. We will restrict attention to k such that the enslaver profits from pursuing and recapturing only high types. It follows immediately that he will pursue any runaway who demonstrated high productivity, $\sigma^*(c_h) = 1$. To illustrate, note that for costs $k > (\bar{q} - \underline{q})(c_h - \gamma)$, the enslaver never pursues anyone, because it is too costly to chase even a high-skilled laborer. While if costs $k = 0$, the enslaver is indifferent to pursuing and recapturing even low-skilled laborers. Thus, let $k \in (0, \bar{k})$, where $\bar{k} = (\bar{q} - \underline{q})(c_h - \gamma)$.

7. The slave's expected utility from escaping may be thought of as her reservation utility, as is the approach of Chwe (1990).

$$U_M = \begin{cases} 2(c_S - \gamma(c_S)) & \text{, if } e_S = 0 \\ c_S - \gamma(c_S) + (1-\bar{q})(c_\theta - \gamma(c_\theta)) & \text{, if } e_S = 1,\, \sigma = 0 \\ c_S - \gamma(c_S) + (1-\underline{q})(c_\theta - \gamma(c_\theta)) - k & \text{, if } e_S = 1,\, \sigma = 1 \end{cases} \tag{1}$$

$$U_S = \begin{cases} \gamma(c_S) & \text{, if } e_S = 0 \\ \gamma(c_S) + (\bar{q})(w_\theta) + (1-\bar{q})(\gamma(c_\theta) - r) & \text{, if } e_S = 1,\, \sigma = 0 \\ \gamma(c_S) + (\underline{q})(w_\theta) + (1-\underline{q})(\gamma(c_\theta) - r) & \text{, if } e_S = 1,\, \sigma = 1 \end{cases} \tag{2}$$

The enslaver's optimal pursuit strategy σ^* is that which maximizes his expected utility, such that $U_M(\sigma^*|\mu,\gamma) \geq U_M(\sigma|\mu,\gamma) \forall \sigma \in [0,1]$.

Assumption 1. Cost $k < \bar{k}$.

If the enslaver observes high output, he knows that the laborer is a high type. If he observes the enslaved laborer produce low and stay, he knows she is a low type, because that strategy is strictly dominated for high types by the strategy to produce high and stay, given $\gamma > 0$. Equation 3 summarizes the values of enslaver M's beliefs for all S_θ strategies.

$$\mu(c_S, e_S) = \begin{cases} 1 & \text{if } c_S = c_h \\ 0 & \text{if } c_S = c_l \text{ and } e_S = 0 \\ \frac{(1-\psi)\cdot\rho}{(1-\psi)\cdot\rho + \pi_l\cdot(1-\rho)} & \text{if } c_S = c_l \text{ and } e_S = 1 \end{cases} \tag{3}$$

Enslaved laborer S_l's strategy is π_l and S_h's strategy is $(\psi, \pi_h(c_S))$, where π_θ is the probability S_θ runs and ψ is the probability S_h produces c_h. High types consecutively choose how much to produce in time $t = 1$ and whether to run in time $t = 2$. Given their production strategy ψ in time $t = 1$, high types subsequently run with the probability $\pi_h(c_S)$ that maximizes their expected utility from that point, such that $U_h(\pi_h^*|c_S,\sigma^*) \geq U_h(\pi_h|c_S,\sigma^*) \forall \pi_h \in [0,1]$. Low types similarly run with probability π_l that maximizes their expected utility, such that $U_l(\pi_l^*|\sigma^*(\alpha_l)) > U_l(\pi_l|\sigma^*(\alpha_l)) \forall \pi_l \in [0,1]$.

Low types always produce the low output c_l by assumption. High types choose between low and high output, given their strategy $\pi_h(c_S)$ and the anticipated enslaver's strategies $\sigma(c_h) = 1$ and $\sigma(c_l) \in [0,1]$. High types produce high if $U_h(\psi = 1, \pi_h(c_h)|\sigma(c_h) = 1) \geq U_h(\psi = 0, \pi_h(c_l)|\sigma(c_l))$.

6 Escape and pursuit

To characterize the perfect Bayesian equilibria, I first consider the enslaver's pursuit strategy. For $k \in (0, \bar{k})$, enslaver M only pursues if he is sufficiently confident that the fugitive is highly-skilled. If the fugitive produced high output in the first time period, the enslaver knows the fugitive is of high skill, because only highly skilled laborers can produce the high amount. So if the enslaver observes high output, he will always pursue. But because it is not profitable for the enslaver to pursue a fugitive of low skill, a highly-skilled laborer might want to mimic the behavior of a low type in order to have a higher chance of successful escape. Since both high and low types might produce the low amount in time period one and escape in the second time period, the enslaver is uncertain if pursuing a fugitive who produced a low amount (hereafter, "low-producing fugitive") is ultimately profitable.

Lemma 1. *In any PBE, the enslaver pursues after observing output c_l if $\mu(c_l, e_S = 1) > \frac{k}{(\bar{q}-\underline{q})(c_h-\gamma)}$.*[8]

Lemma 2. *There does not exist an equilibrium in which the enslaver always pursues.*[9]

8. See derivation in appendix A.

9. See appendix B.

Equilibria in which the enslaver never pursues after observing low output, $\sigma(c_l) = 0$.

Case 1: $w_l \leq \frac{r(1-\bar{q})}{\bar{q}}$

If the outside wage for low-skilled laborers does not incentivize S_l to escape, specifically, when $w_l < \frac{r(1-\bar{q})}{\bar{q}}$, then enslaved laborers of high type cannot conceal by producing the low amount. In other words, enslaver M would know that anyone who ran was of type $\theta = h$, and he would thus always pursue after observing output c_l. In that case, the high type does not benefit from concealing, and so if she chooses to escape, she does so after revealing.

Proposition 1. *For* $w_l \leq \frac{r(1-\bar{q})}{\bar{q}}$,

a) *there exists a PBE in which low types stay, high types produce high and run, and the enslaver pursues runaways if and only if they produced high when* $w_h > \gamma + \frac{r(1-\underline{q})}{\underline{q}}$; *and*

b) *there exists a PBE in which both types stay, high types produce high, and the enslaver pursues runaways if and only if they produced high when* $w_h < \gamma + \frac{r(1-\underline{q})}{\underline{q}}$.[10]

Case 2: $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$

Alternatively, if low types prefer to run than to stay, high types face a tradeoff between revealing and receiving the higher reward in time period 1 or concealing and gaining a higher probability of successful escape in time $t = 2$.

If the plantation reward is sufficiently low, $\gamma < r(\frac{\bar{q}}{\underline{q}} - 1) := \hat{\gamma}$, it is never worthwhile for slave S_h to reveal in the first time period. Figure 1(a) depicts the range of outside wages given this low γ. Let B be the outside wage that makes S_h indifferent between concealing and staying, and running. If $w_h < B$, then S_h's utility-maximizing strategy is to stay and produce amount c_h. If $w_h > B$, then her optimal choice is to conceal and run. In equilibrium, enslavers do not pursue if

10. See appendix C.1.

they believe the runaway is sufficiently unlikely to be of high type, $\mu(c_S, e_S) < \frac{k}{(\bar{q}-\underline{q})(c_h-\gamma)}$, where $\mu(c_S, e_S) = \frac{(1-\psi)\rho}{(1-\psi)\cdot\rho+\pi_l(1-\rho)}$.

Proposition 2. *For $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$ and $\gamma < \hat{\gamma}$,*

a) there exists a PBE in which low types run, high types produce high and stay, and the enslaver, believing with certainty that those who produce the low amount are low types, pursues runaways if and only if they produced high when $w_h < B$; and

b) there exists a PBE in which both types produce low and run, and the enslaver, believing it is sufficiently unlikely that low-producing fugitives are in fact high types, $\rho < \frac{k}{\bar{k}}$, pursues runaways if and only if they produced high when $w_h > B$.[11]

If the plantation reward is sufficiently high, $\gamma > \hat{\gamma}$, certain outside wages incentivize S_h to reveal and run, being pursued with certainty. Figure 1(b) depicts the range of wages and S_h's corresponding preferences for this high γ. For the lowest wages, $w_h < A$, enslaved laborer S_h prefers to stay and produce the high output. For wages greater than A but less than C, S_h profits from revealing and running. For the most advantageous outside options, $w_h > C$, laborer S_h forgoes reward γ in exchange for higher probability of escape. Again, however, this is only correct when, in equilibrium, the enslaver believes she is sufficiently unlikely to be of high type, $\mu(c_S, e_S) < \frac{k}{(\bar{q}-\underline{q})(c_h-\gamma)}$.

11. See appendix C.2.1.

Proposition 3. *For $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$ and $\gamma > \hat{\gamma}$,*

a) there exists a PBE in which low types run, high types produce high and stay, and the enslaver, believing with certainty that those who produce the low amount are low types, pursues runaways if and only if they produced high when $w_h < A$; and

b) there exists a PBE in which both types produce low and run, and the enslaver, believing it is sufficiently unlikely that low-producing fugitives are in fact high types, $\rho < \frac{k}{\bar{k}}$, pursues runaways if and only if they produced high when $w_H > C$; and

c) there exists a PBE in which low types run, high types produce high and run, and the enslaver, believing with certainty that those who produce the low amount are low types, pursues runaways if and only if they produced high when $C > w_h > A$.[12]

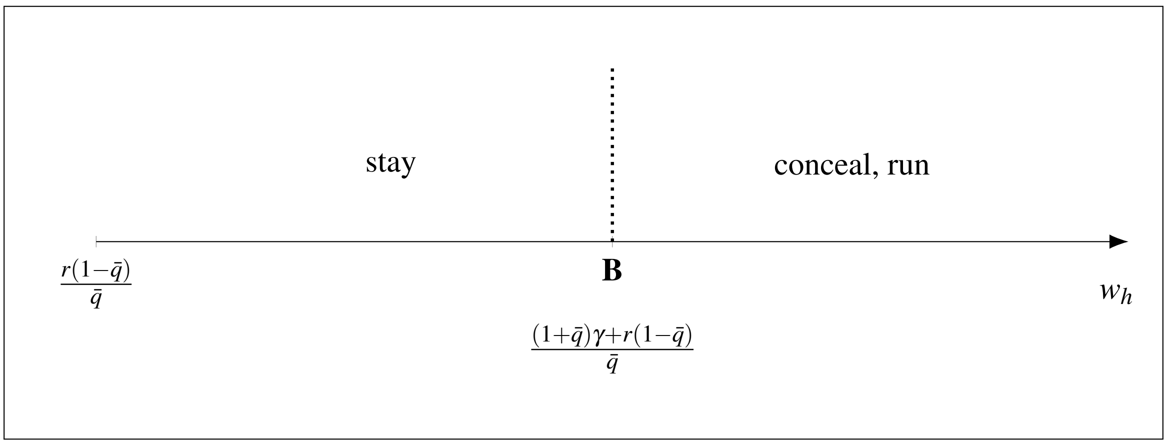
Equilibria in which M pursues after observing high output and is indifferent between pursuing and not after observing low output, $\sigma(c_h) = 1$ and $\sigma(c_l) \in (0,1)$.

Recall that $\mu(c_S, e_S)$ is enslaver M's posterior belief, after observing the amount produced and an escape attempt, that the laborer is a high type. If enslaver M is indifferent between pursuing and not, then $k = \hat{\mu}(c_l, e_S = 1) \cdot (\bar{q} - \underline{q})(c_h - \gamma)$, where $\hat{\mu}$ is the posterior belief that makes M indifferent. Supposing M is indifferent, it follows that, in equilibrium, high types weakly or strictly prefer to conceal and run. To see that this must be so, observe that if high types preferred to produce high in the first time period, then M would know that those who produced low were exclusively low types, and so M would never pursue after observing low, $\sigma(c_l) = 0$, $\Rightarrow\Leftarrow$. It must also be that low types weakly or strictly prefer to run, because if low types never ran, then $\sigma(c_l) = 1$, $\Rightarrow\Leftarrow$. See all derivations in appendix D.

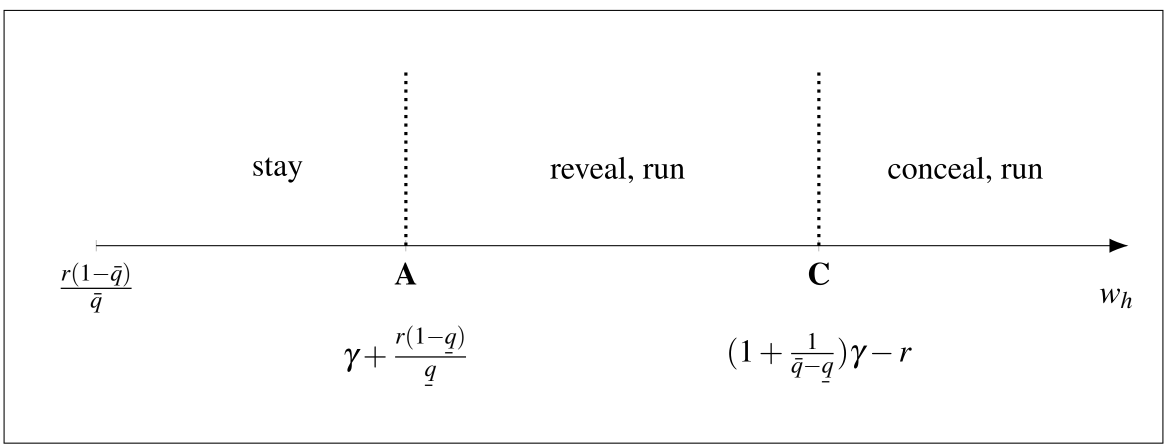
12. See appendix C.2.2.

Figure 1: S_h ideal strategy for various outside wages

(a) $\gamma < \hat{\gamma}, A > B > C$



(b) $\gamma > \hat{\gamma}, C > B > A$



Case 1: $\rho < \hat{\mu}$

To achieve $\mu = \hat{\mu}$, it must be that high types prefer to attempt escape after $c_S = c_l$, and that low types are indifferent between staying and escaping. As such, the belief $\hat{\mu}$ is greater than share of high types in the population ρ.

Low types are indifferent between running and staying when the enslaver pursues with probability

$$\sigma^* = \frac{\bar{q} \cdot w_l - (1-\bar{q})r}{(\bar{q} - \underline{q})(w_l + r)}$$

For sufficiently high outside wages, high types strictly prefer to conceal and run, given σ^*.

Proposition 4. *For $\rho < \frac{k}{\bar{k}}$ and $w_l \in (\frac{r(1-\underline{q})}{\underline{q}}, \frac{r(1-\bar{q})}{\bar{q}})$, $\exists$ a value $\hat{w}_h$ such that $\forall w_h > \hat{w}_h$,*

there exists a PBE in which high types produce low and run, and low types run with probability $\pi_l^* = (\bar{k} - k)\frac{\rho}{1-\rho}$; the enslaver pursues runaways that produced high and pursues those that produced low with probability σ^*, believing that low-producing fugitives are in fact high types with probability $\hat{\mu}$.[13]

Case 2: $\rho > \hat{\mu}$

To achieve $\mu = \hat{\mu}$, it must be that low types always run, while only some high types conceal and run. High types are indifferent between concealing and running, and not concealing their type when $U_h(\psi = 0, \pi(c_l) = 1 | \sigma(c_l) \in (0,1)) = \max\{U_h(\psi = 1, \pi(c_h) = 1 | \sigma(c_l) \in (0,1)), U_h(\psi = 1, \pi(c_h) = 0 | \sigma(c_l) \in (0,1))\}$.

Consider the case when $U_h(\psi = 1, \pi(c_h) = 0 | \sigma(c_l) \in (0,1)) > U_h(\psi = 1, \pi(c_h) = 1 | \sigma(c_l) \in (0,1))$. Then, high types are indifferent between concealing and running and producing high and staying when

$$\hat{\sigma}_1 = \frac{\gamma + r + \bar{q}(\gamma - r - w_h)}{(\bar{q} - \underline{q})(\gamma - r - w_h)}$$

13. See appendix D.1.

When the outside wage for low-skilled laborers is sufficiently close to that for high-skilled laborers, low types strictly prefer to run, given $\hat{\sigma}_1$. High types then conceal with probability

$$1-\psi^* = \frac{(1-\rho)\cdot k}{\rho\cdot((\bar{q}-\underline{q})(c_h-\gamma)-k)}$$

and subsequently run, $\pi_h^*(c_l)=1$.

Consider next the case when $U_h(\psi=1,\pi(c_h)=1|\sigma(c_l)\in(0,1)) > U_h(\psi=1,\pi(c_h)=0|\sigma(c_l)\in(0,1))$. Then, high types are indifferent between concealing and running and revealing and running when

$$\hat{\sigma}_2 = \frac{\gamma}{(\bar{q}-\underline{q})(\gamma-w_h-r)}+1$$

Low types always run, given the constraints for $\hat{\sigma}_2\in(0,1)$, and high types conceal with probability $1-\psi^*$ and subsequently run, $\pi_h^*(c_l)=1$.

Proposition 5. *For $\rho > \frac{k}{\bar{k}}$,*

a) there exists a PBE in which low types run and high types produce low with probability $\frac{\rho\cdot\bar{k}-k}{\rho(\bar{k}-k)}$ and run; the enslaver, believing that low-producing fugitives are in fact high types with probability $\hat{\mu}$, pursues runaways that produced high and pursues those that produced low with probability $\hat{\sigma}_1$ when $w_h\in(\frac{w_l\cdot\gamma}{r}+2\gamma+w_l, \frac{\gamma(1+\underline{q})+r(1-\underline{q})}{\underline{q}})$; and

b) there exists a PBE in which low types run and high types produce low with probability $\frac{\rho\cdot\bar{k}-k}{\rho(\bar{k}-k)}$ and run; the enslaver, believing that low-producing fugitives are in fact high types with probability $\hat{\mu}$, pursues runaways that produced high and pursues those that produced low with probability $\hat{\sigma}_2$ when $w_h > \frac{\gamma}{\bar{q}-\underline{q}}+\gamma-r$.[14]

Case 3: $\rho=\hat{\mu}$

If both types produce low and run, or do so in equal proportions, i.e., $1-\psi^*=\pi_l^*$, then the enslaver's posterior belief is $\mu(c_l,e_S)=\rho=\hat{\mu}$, and the share of high types in the population is $\frac{k}{\bar{k}}$.

14. See appendix D.2.

Let $\sigma^{\dagger}$ be the probability of pursuit that makes both types prefer to produce low and run. Both types strictly prefer to produce low and run when $\sigma^{\dagger} := \sigma < \hat{\sigma}_2$ because $\hat{\sigma}_2 = \min\{\sigma^*, \hat{\sigma}_1, \hat{\sigma}_2\}$.

Proposition 6. *For $\rho = \frac{\underline{k}}{\bar{k}}$,*

> *there exists a PBE in which both types produce low and run, and the enslaver, believing that low-producing fugitives are in fact high types with probability $\hat{\mu} = \rho$, pursues runaways that produced high and pursues those that produced low with probability $\sigma^{\dagger}$ when $w_h > \frac{\gamma}{\bar{q}-\underline{q}} + \gamma - r$.*[15]

7 Extension, γ selection

In order to understand the enslaver's potential tradeoff between offering higher rewards and investing in recapture, I endogenize the reward premium γ.

Extension set-up

As before, the game begins with Nature determining the type θ of enslaved person S.

Let the game proceed with enslaver M determining the reward schedule γ for low and high outputs c_l and c_h. Again,

$$\gamma(c_S) = \begin{cases} \gamma & \text{, if } c_S \geq c_h \\ 0 & \text{, if } c_S \in [c_l, c_h), \end{cases}$$

where reward $\gamma \in (0, c_h)$. We assume commitment.[16] The game proceeds as in the baseline model, and payoffs are the same.

15. See appendix D.3.

16. The assumption of commitment is substantively plausible for several reasons. As discussed in section 3, there is ample anecdotal evidence that enslavers in fact traded rewards for productivity. This is also Fogel and Engerman (1974b)'s theoretical and empirical conclusion. Finally, because enslavers were employing and exploiting the enslaved for many years, they benefited from establishing a credible reputation if they wanted to extract the reward-incentivized effort more than once.

Extension solution

Enslaver M determines the amount γ that maximizes his profit given his pursuit strategy and enslaved laborers' preferences over production and running. Specifically, he chooses γ^* such that $U_M(\gamma^*|\sigma^*(\gamma^*), \pi^*_\theta(\gamma^*), \psi^*(\gamma^*)) \geq U_M(\gamma|\sigma^*(\gamma), \pi^*_\theta(\gamma), \psi^*(\gamma)) \forall \gamma \in (0, c_h)$.

Equilibrium specification TBD

8 Conclusion

To understand the US South, one has to understand how it maintained the antebellum slavocracy. Elites expended private resources and employed public institutions in order to curb the threat of escape by enslaved persons. My model highlights that the decisions of enslaver and enslaved on the plantations were functions of the external repressive environment. The results provide theoretical motivation for future studies of runaway slave ads and of antebellum elites' demands on the state.

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Appendices

A Enslaver belief

Recall that $\mu(c_S, e_S)$ is enslaver M's belief that enslaved laborer S_θ is of type $\theta = h$, which he updates after observing whether S_θ produces amount c_h and runs, $e_S = 1$.

$$\mu(c_S, e_S) = \frac{Pr(c_S, e_S|\theta = h) \cdot \rho}{Pr(c_S, e_S|\theta = h) \cdot \rho + Pr(c_S, e_S|\theta = l) \cdot (1-\rho)}$$

If M observes output c_l, he prefers to pursue S_θ if

$$\begin{aligned}
U_M(\sigma(c_l) = 1|c_l, e_S = 1) > & \quad U_M(\sigma(c_l) = 0|c_l, e_S = 1) \\
\mu(c_l, e_S = 1)(1-\underline{q})(c_h - \gamma) - k > & \quad \mu(c_l, e_S = 1)(1-\bar{q})(c_h - \gamma) \\
\mu(c_l, e_S = 1)(1-\underline{q} - 1 + \bar{q})(c_h - \gamma) > & \quad k \\
\mu(c_l, e_S = 1)(\bar{q} - \underline{q})(c_h - \gamma) > & \quad k \\
\mu(c_l, e_S = 1)\bar{k} > & \quad k
\end{aligned}$$

Belief $\mu(c_l, e_S = 1) < 1$ because $\bar{k} > k$. If enslaver M observes that S_θ produced low and ran,

$$\mu(c_l, e_S = 1) = \frac{(1-\psi)\rho}{(1-\psi) \cdot \rho + \pi_l(1-\rho)}$$

B Proof, $\sigma(c_l) \neq 1$

Suppose enslaver M pursues after observing both low and high outputs, $\sigma(c_l) = 1$.

Regardless of whether or not low types escape, enslaved laborers of high type prefer to reveal and escape rather than conceal and escape. This is because both strategies result in the same probability of successful escape in time period 2, but revealing provides a higher payoff in time period 1. Realizing that the high type's strategy is to reveal, the enslaver knows with certainty that an enslaved laborer who produces c_l is of low type. Because he only benefits from pursuing high types, he does not pursue after observing low output, $\sigma(c_l) = 0$. $\Rightarrow\Leftarrow$

C $\sigma(c_l) = 0$

C.1 $w_l \leq \frac{r(1-\bar{q})}{\bar{q}}$

Consider the values of the low external wage such that low types never prefer to run, $w_l \leq \frac{r(1-\bar{q})}{\bar{q}}$. Given the condition on w_l, there exists an equilibrium with $\sigma(c_l) = 0$. Low types stay, and high types run if $\gamma < w_h - \frac{r(1-\bar{q})}{\bar{q}}$.

The equilibrium strategy profiles are as follows, and mirror the presentation in proposition 1.

For $w_l \leq \frac{r(1-\bar{q})}{\bar{q}}$,

a) $(\pi_l^* = 0,\ \psi^* = 1,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 1,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = 0)$ when $w_h > \gamma + \frac{r(1-\underline{q})}{\underline{q}}$

b.1) $(\pi_l^* = 0,\ \psi^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = 0)$ when $w_h \in (\gamma + \frac{r(1-\bar{q})}{\bar{q}}, \gamma + \frac{r(1-\underline{q})}{\underline{q}})$

b.2) $(\pi_l^* = 0,\ \psi^* = 1,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = 0)$ when $w_h < \gamma + \frac{r(1-\bar{q})}{\bar{q}}$

The high type's strategy $\pi_h^*(c_l)$ is off the path of play. Recall that high types, if they stay, always prefer to produce high in the second time period, regardless of their first-period production decision. If S_h chose c_l in time $t = 1$, she would be choosing in time $t = 2$ between γ and $\bar{q} \cdot w_h + (1-\bar{q})(\gamma - r)$. She never chooses c_l in time $t = 2$, because doing so is strictly dominated by choosing c_h in time $t = 2$. If $w_h < \gamma + \frac{r(1-\bar{q})}{\bar{q}}$, she prefers to stay, $\pi_h^*(c_l) = 0$, and otherwise she prefers to run, $\pi_h^*(c_l) = 1$.

C.2 $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$

Consider the values of the low external wage such that low types always prefer to run, $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$.

High types prefer to stay rather than produce high and run when the outside wage w_h is less than $A = \gamma + \frac{r(1-\underline{q})}{\underline{q}}$.

$$\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 0 | \sigma(c_l) = 0) > & \quad U_h(\psi = 1, \pi(c_h) = 1 | \sigma(c_l) = 0) \\
2\gamma > & \quad \gamma + \underline{q} \cdot w_h + (1-\underline{q})(\gamma - r) \\
\underline{q} \cdot \gamma > & \quad \underline{q} \cdot w_h - (1-\underline{q})r \\
w_h < & \quad \gamma + \frac{r(1-\underline{q})}{\underline{q}}
\end{aligned}$$

High types prefer to stay rather than conceal and run when the outside wage w_h is less than $B = \frac{(1+\bar{q})\gamma + r(1-\bar{q})}{\bar{q}}$.

$$\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 0 | \sigma(c_l) = 0) > & \quad U_h(\psi = 0, \pi(c_l) = 1 | \sigma(c_l) = 0) \\
2\gamma > & \quad \bar{q} \cdot w_h + (1-\bar{q})(\gamma - r) \\
w_h < & \quad \frac{(1+\bar{q})\gamma + r(1-\bar{q})}{\bar{q}}
\end{aligned}$$

High types prefer to stay produce high and run than to conceal and run if the outside wage w_h is less than $C = (1 + \frac{1}{\bar{q}-\underline{q}})\gamma - r$.

$$\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 1 | \sigma(c_l) = 0) > & \quad U_h(\psi = 0, \pi(c_l) = 1 | \sigma(c_l) = 0) \\
\gamma + \underline{q} \cdot w_h + (1-\underline{q})(\gamma - r) > & \quad \bar{q} \cdot w_h + (1-\bar{q})(\gamma - r) \\
w_h < & \quad (1 + \frac{1}{\bar{q}-\underline{q}})\gamma - r
\end{aligned}$$

Table 1 summarizes the above values.

In order to compare the utilities in terms of the outside wage, I compare A, B, C.

Table 1: Critical values of wage w_h

A	$U_h(\psi=1,\pi=0\vert\sigma(c_l)=0)=U_h(\psi=1,\pi=1\vert\sigma(c_l)=0)$	$\gamma+\frac{r(1-\underline{q})}{\underline{q}}$
B	$U_h(\psi=0,\pi=1\vert\sigma(c_l)=0)=U_h(\psi=1,\pi=0\vert\sigma(c_l)=0)$	$\frac{(1+\bar{q})\gamma+r(1-\bar{q})}{\bar{q}}$
C	$U_h(\psi=1,\pi=1\vert\sigma(c_l)=0)=U_h(\psi=0,\pi=1\vert\sigma(c_l)=0)$	$(1+\frac{1}{\bar{q}-\underline{q}})\gamma-r$

$$\begin{aligned}
A &> B\\
\gamma+\frac{r(1-\underline{q})}{\underline{q}} &> \frac{(1+\bar{q})\gamma+r(1-\bar{q})}{\bar{q}}\\
\gamma\cdot\bar{q}+\frac{r\cdot\bar{q}(1-\underline{q})}{\underline{q}} &> (1+\bar{q})\gamma+r(1-\bar{q})\\
\frac{r\cdot\bar{q}(1-\underline{q})}{\underline{q}} &> \gamma+r(1-\bar{q})\\
r[\frac{\bar{q}(1-\underline{q})}{\underline{q}}-1+\bar{q}] &> \gamma\\
r(\frac{\bar{q}}{\underline{q}}-\bar{q}-1+\bar{q}) &> \gamma\\
r(\frac{\bar{q}}{\underline{q}}-1) &> \gamma
\end{aligned}$$

$$\begin{aligned}
A &> C\\
\gamma+\frac{r(1-\underline{q})}{\underline{q}} &> (1+\frac{1}{\bar{q}-\underline{q}})\gamma-r\\
r(1-\underline{q}) &> \frac{\gamma\cdot\underline{q}}{\bar{q}-\underline{q}}-r\cdot\underline{q}\\
\frac{r(\bar{q}-\underline{q})}{\underline{q}} &> \gamma\\
r(\frac{\bar{q}}{\underline{q}}-1) &> \gamma
\end{aligned}$$

$$
\begin{aligned}
B > & \quad C \\
\frac{(1+\bar{q})\gamma + r(1-\bar{q})}{\bar{q}} > & \quad (1+\frac{1}{\bar{q}-\underline{q}})\gamma - r \\
\gamma - r + \frac{\gamma + r}{\bar{q}} > & \quad \gamma + \frac{\gamma}{\bar{q}-\underline{q}} - r \\
\frac{\gamma + r}{\bar{q}} > & \quad \frac{\gamma}{\bar{q}-\underline{q}} \\
\gamma + r > & \quad \frac{\gamma \cdot \bar{q}}{\bar{q}-\underline{q}} \\
r > & \quad \gamma(\frac{\bar{q}}{\bar{q}-\underline{q}} - 1) \\
r > & \quad \gamma \cdot \frac{\bar{q} - \bar{q} + \underline{q}}{\bar{q}-\underline{q}} \\
r > & \quad \gamma \cdot \frac{\underline{q}}{\bar{q}-\underline{q}} \\
\frac{r(\bar{q}-\underline{q})}{\underline{q}} > & \quad \gamma \\
r(\frac{\bar{q}}{\underline{q}} - 1) > & \quad \gamma
\end{aligned}
$$

Let $\hat{\gamma} := r(\frac{\bar{q}}{\underline{q}} - 1)$. If $\gamma < \hat{\gamma}$, $A > B > C$, and if $\gamma > \hat{\gamma}$, $C > B > A$.

If $\gamma > \hat{\gamma}$ and $w_H < A$ or $\gamma < \hat{\gamma}$ and $w_h < B$, high types prefer to stay. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = 0$.

$$
\begin{aligned}
\mu(c_l, e_S = 1) = & \quad \frac{(1-\psi)\rho}{(1-\psi)\rho + \pi_l(1-\rho)} \\
\mu(c_l, e_S = 1) = & \quad \frac{(1-1)\rho}{(1-1)\rho + 1(1-\rho)} \\
\mu(c_l, e_S = 1) = & \quad 0
\end{aligned}
$$

The enslaver therefore never wants to pursue after observing low output, because $\mu \cdot (\bar{q} - \underline{q})(c_h - \gamma) \ngeq k$.

If $\gamma > \hat{\gamma}$ and $w_H > C$ or $\gamma < \hat{\gamma}$ and $w_h > B$, high types prefer to conceal and run. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = \rho$.

$$\mu(c_l, e_S = 1) = \frac{(1-\psi)\rho}{(1-\psi)\rho + \pi_l(1-\rho)}$$
$$\mu(c_l, e_S = 1) = \frac{(1-0)\rho}{(1-0)\rho + 1(1-\rho)}$$
$$\mu(c_l, e_S = 1) = \rho$$

Still, the enslaver does not benefit from pursuing after observing low output if $k > \rho \cdot (\bar{q} - \underline{q})(c_h - \gamma)$. Therefore $\rho < \frac{k}{k}$ can sustain an equilibrium in which both high and low types produce low and run, and the enslaver does not pursue, given the stated external wages.

If $\gamma > \hat{\gamma}$ and $C > w_h > A$, then high types prefer to reveal and run. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = 0$.

$$\mu(c_l, e_S = 1) = \frac{(1-\psi)\rho}{(1-\psi)\rho + \pi_l(1-\rho)}$$
$$\mu(c_l, e_S = 1) = \frac{(1-1)\rho}{(1-1)\rho + 1(1-\rho)}$$
$$\mu(c_l, e_S = 1) = 0$$

As above, the enslaver never wants to pursue after observing low output, because $\mu \cdot (\bar{q} - \underline{q})(c_h - \gamma) \ngeq k$.

C.2.1 $\gamma < \hat{\gamma}$

The equilibrium strategy profiles when $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$ and $\gamma < \hat{\gamma}$ are as follows. This mirrors the presentation in proposition 2.

For $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$ and $\gamma < \hat{\gamma}$

a.1) $(\pi_l^* = 1,\ \psi^* = 1,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = 0)$ when $w_h < \gamma - r(\frac{1-\bar{q}}{q})$

a.2) $(\pi_l^* = 1,\ \psi^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = 0)$ when $\gamma - r(\frac{1-\bar{q}}{q}) < w_h < B$

b.1) $(\pi_l^* = 1,\ \psi^* = 0,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = \rho)$ when $B < w_h < A$ and $\rho < \frac{k}{\bar{k}}$

b.2) $(\pi_l^* = 1,\ \psi^* = 0,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 1,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = \rho\ )$ when $A < w_h$ and $\rho < \frac{k}{\bar{k}}$

The high type's strategy $\pi_h^*(c_l)$ is off the path of play when $w_h < B$. If S_h chose c_l in time $t = 1$, given $w_h < B$, she would be choosing in time $t = 2$ between γ and $\bar{q} \cdot w_h + (1-\bar{q})(\gamma - r)$. She prefers to run, $\pi_h^*(c_l) = 1$, if $w_h > \gamma - r(\frac{1-\bar{q}}{q})$. Note that $B > \gamma - r(\frac{1-\bar{q}}{q})$.

When $w_h > B$, the strategy $\pi_h^*(c_h)$ is off the path of play. If S_h chose c_h in time $t = 1$, given $w_h > B$, she would prefer to stay when $B < w_h < A$, and she would prefer to run if $w_h > A$.

C.2.2 $\gamma > \hat{\gamma}$

The equilibrium strategy profiles when $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$ and $\gamma > \hat{\gamma}$ are as follows. This mirrors the presentation in proposition 3.

For $w_l \geq \frac{r(1-\bar{q})}{\bar{q}}$ and $\gamma > \hat{\gamma}$

a.1) $(\pi_l^* = 1,\ \psi^* = 1,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1\ ,\ \mu = 0)$ when $w_h < \gamma + r(\frac{1-\bar{q}}{\bar{q}})$

a.2) $(\pi_l^* = 1,\ \psi^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1\ ,\ \mu = 0)$ when $\gamma + r(\frac{1-\bar{q}}{\bar{q}}) < w_h < A$

b) $(\pi_l^* = 1,\ \psi^* = 0,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 1,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1\ ,\ \mu = \rho)$ when $w_h > C$ and $\rho < \frac{k}{\bar{k}}$

c) $(\pi_l^* = 1,\ \psi^* = 1,\ \pi_h^*(c_l) = \pi_h^*(c_h) = 1,\ \sigma^*(c_l) = 0,\ \sigma^*(c_h) = 1,\ \mu = 0)$ when $C > w_h > A$.

When $w_h < C$, the high type's strategy $\pi_h^*(c_l)$ is off the path of play. If S_h chose c_l in time $t = 1$, she prefers to stay if $w_h < \gamma + r(\frac{1-\bar{q}}{\bar{q}})$. Note that $A > \gamma + r(\frac{1-\bar{q}}{\bar{q}}$.

When $w_h > C$, the high type's strategy $\pi_h^*(c_h)$ is off the path of play. If S_h chose c_h in time $t = 1$, she wants to run because $w_h > \gamma + r(\frac{1-\underline{q}}{\underline{q}})$. Note that $C > \gamma + r(\frac{1-\underline{q}}{\underline{q}})$.

D $\sigma(c_l) \in (0,1)$

Enslaver M is indifferent between pursuing and not when $k = \mu(c_l, e_S = 1) \cdot (\bar{q} - \underline{q})(c_h - \gamma)$.

D.1 Suppose S_l is indifferent and S_h conceals and runs.

Let σ^* be the probability of pursuit that makes S_l indifferent between staying and running.

$$\begin{aligned} 0 = &\quad \sigma(\underline{q} \cdot w_l - (1-\underline{q})r) + (1-\sigma)(\bar{q} \cdot w_l - (1-\bar{q})r) \\ \sigma[(\bar{q} \cdot w_l - (1-\bar{q})r) - (\underline{q} \cdot w_l - (1-\underline{q})r)] = &\quad \bar{q} \cdot w_l - (1-\bar{q})r \\ \sigma^* = &\quad \frac{\bar{q} \cdot w_l - (1-\bar{q})r}{(\bar{q} - \underline{q})(w_l + r)} \end{aligned}$$

The denominator is always positive, and the numerator is positive when $\frac{r(1-\underline{q})}{\underline{q}} > 0$. The denominator is greater than the numerator when $\frac{r(1-\underline{q})}{\underline{q}} > w_l$. So, $\sigma^* \in (0,1)$ when $\frac{r(1-\underline{q})}{\underline{q}} > w_l > \frac{r(1-\bar{q})}{\bar{q}}$.

When the enslaver pursues with probability σ^*, the high type's utility from running is as follows.

$$\sigma^*(\gamma - r - w_h)(\bar{q} - \underline{q}) + \bar{q}\cdot w_h - (1-\bar{q})(\gamma - r)$$
$$\frac{(\bar{q}\cdot w_l - r + \bar{q}\cdot r)(\gamma - r - w_h)}{w_l + r} + \bar{q}\cdot w_h - (1-\bar{q})(\gamma - r)$$
$$\frac{(\bar{q}\cdot w_l - r + \bar{q}\cdot r)(\gamma - r - w_h) + (w_l + r)(\bar{q}\cdot w_h - \gamma + r + \bar{q}\cdot\gamma - \bar{q}\cdot r)}{w_l + r}$$
$$\frac{2(\bar{q}\cdot w_l\cdot(\gamma - r) + r\cdot\gamma\cdot(\bar{q}-1) + r^2(1-\bar{q})) + r(w_h + w_l - \gamma)}{w_l + r}$$

She prefers to conceal and run than to stay when the outside wage is sufficiently high, specifically, $w_h > (5-2\bar{q})\gamma + \frac{(2-2\bar{q})(\gamma\cdot w_l - r^2)}{r} + (2\bar{q}-1)w_l$.

$$\begin{aligned}
\frac{2(\bar{q}\cdot w_l(\gamma - r) + r\cdot\gamma(\bar{q}-1) + r^2(1-\bar{q})) + r(w_h + w_l - \gamma)}{w_l + r} &> 2\gamma \\
2(\bar{q}\cdot w_l(\gamma - r) + r\cdot\gamma(\bar{q}-1) + r^2(1-\bar{q})) + r(w_h + w_l - \gamma) &> 2\gamma(w_l + r) \\
2(\bar{q}\cdot w_l(-r) + r^2(1-\bar{q})) + r(w_h + w_l) &> (5-2\bar{q})\gamma\cdot r + (2-2\bar{q})\gamma\cdot w_l \\
(1-2\bar{q})w_l + w_h &> (5-2\bar{q})\gamma + \frac{(2-2\bar{q})(\gamma\cdot w_l - r^2)}{r}
\end{aligned}$$

Recalling that $\sigma^* \in (0,1)$ when $\frac{r(1-\underline{q})}{\underline{q}} > w_l > \frac{r(1-\bar{q})}{\bar{q}}$, the high type prefers concealing to revealing when the outside wage is sufficiently high.

$$w_h > \frac{(\underline{q} + 2\bar{q} - 1)\gamma\cdot w_l - (4 - \underline{q} - 2\bar{q})\gamma\cdot r + (2 - 2\bar{q} - \underline{q})w_l\cdot r + (3 - \underline{q} - 2\bar{q})r^2}{\underline{q}\cdot w_l - r(1-\underline{q})}$$

See derivation below.

$$\begin{aligned}
\frac{2(\bar{q}\cdot w_l(\gamma - r) + r\cdot\gamma(\bar{q}-1) + r^2(1-\bar{q})) + r(w_h + w_l - \gamma)}{w_l + r} &> \underline{q}w_h + (1-\underline{q})(\gamma - r) \\
2(\bar{q}\cdot w_l(\gamma - r) + r\cdot\gamma(\bar{q}-1) + r^2(1-\bar{q})) + r(w_h + w_l - \gamma) &> (w_l + r)(\underline{q}w_h + (1-\underline{q})(\gamma - r) \\
(\underline{q} + 2\bar{q} - 1)\gamma\cdot w_l - (4 - \underline{q} - 2\bar{q})\gamma\cdot r + (2 - 2\bar{q} - \underline{q})w_l\cdot r + (3 - \underline{q} - 2\bar{q})r^2 &> w_h(\underline{q}\cdot w_l - r(1-\underline{q}))
\end{aligned}$$

Given that the enslaver is indifferent between pursuing and not, we can solve for the probability that S_l runs, π_l.

$$\begin{aligned}
\frac{(1-\psi)\rho}{(1-\psi)\rho+\pi_l(1-\rho)}\cdot(\bar{q}-\underline{q})(c_h-\gamma) &= && k\\
\frac{\rho}{\rho+\pi_l(1-\rho)}\cdot(\bar{q}-\underline{q})(c_h-\gamma) &= && k\\
\rho\cdot(\bar{q}-\underline{q})(c_h-\gamma) &= && k(\rho+\pi_l(1-\rho))\\
((\bar{q}-\underline{q})(c_h-\gamma)-k)\frac{\rho}{1-\rho} &= && \pi_l^*
\end{aligned}$$

Probability $\pi_l^* < (0,1)$ when $\rho < \frac{1}{1+\bar{k}-k}$, which is always the case when $\rho < \frac{k}{\bar{k}}$.

Let $\hat{w_h}$ be defined as follows.

$$\begin{aligned}
w_h > \max\{&(5-2\bar{q})\gamma+\frac{(2-2\bar{q})(\gamma\cdot w_l-r^2)}{r}+(2\bar{q}-1)w_l,\\
&\frac{(\underline{q}+2\bar{q}-1)\gamma\cdot w_l-(4-\underline{q}-2\bar{q})\gamma\cdot r+(2-2\bar{q}-\underline{q})w_l\cdot r+(3-\underline{q}-2\bar{q})r^2}{\underline{q}\cdot w_l-r(1-\underline{q})}\}
\end{aligned}$$

The equilibrium strategy profile when $\rho < \frac{k}{\bar{k}}$, $w_l \in (\frac{r(1-\underline{q})}{\underline{q}}, \frac{r(1-\bar{q})}{\bar{q}})$, and $w_h > \hat{w}_h$ follows. This mirrors the presentation in proposition 4.

For $\rho < \frac{k}{\bar{k}}$ and $w_l \in (\frac{r(1-\underline{q})}{\underline{q}}, \frac{r(1-\bar{q})}{\bar{q}})$,

$(\psi^* = 0,\ \pi_l^* = (\bar{k}-k)\frac{\rho}{1-\rho},\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \sigma^*(c_l) = \sigma^*,\ \sigma^*(c_h) = 1,\ \mu = \hat{\mu})$
when $w_h > (5-2\bar{q})\gamma+\frac{(2-2\bar{q})(\gamma\cdot w_l-r^2)}{r}+(2\bar{q}-1)w_l$

$(\psi^* = 0,\ \pi_l^* = (\bar{k}-k)\frac{\rho}{1-\rho},\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \sigma^*(c_l) = \sigma^*,\ \sigma^*(c_h) = 1,\ \mu = \hat{\mu})$
when $w_h > \frac{(\underline{q}+2\bar{q}-1)\gamma\cdot w_l-(4-\underline{q}-2\bar{q})\gamma\cdot r+(2-2\bar{q}-\underline{q})w_l\cdot r+(3-\underline{q}-2\bar{q})r^2}{\underline{q}\cdot w_l-r(1-\underline{q})}$

D.2 Suppose S_h is indifferent and S_l runs.

D.2.1

When the enslaver pursues with probability $\hat{\sigma}_1 = \frac{\gamma+r+\bar{q}(\gamma-r-w_h)}{(\bar{q}-\underline{q})(\gamma-r-w_h)}$, high types are indifferent between staying versus concealing and running.

$$
\begin{aligned}
U_h(\psi=1,\pi(c_h)=0|\sigma(c_l)\in(0,1)) = {} & U_h(\psi=0,\pi(c_l)=1|\sigma(c_l)\in(0,1)) \\
2\gamma = {} & w_h(\sigma\cdot\underline{q}+(1-\sigma)(\bar{q}))+(\gamma-r)(\sigma\cdot(1-\underline{q})+(1-\sigma)(1-\bar{q})) \\
2\gamma-\bar{q}\cdot w_h-(1-\bar{q})(\gamma-r) = {} & \sigma(\gamma-r-w_h)(\bar{q}-\underline{q}) \\
\frac{\gamma+r+\bar{q}(\gamma-r-w_h)}{(\bar{q}-\underline{q})(\gamma-r-w_h)} = {} & \hat{\sigma}_1
\end{aligned}
$$

$\hat{\sigma}_1 \in (0,1)$ when $w_h > \frac{\gamma(1+\underline{q})+r(1-\underline{q})}{\underline{q}}$. Note that when $w_h > \frac{\gamma(1+\underline{q})+r(1-\underline{q})}{\underline{q}}$, $w_h > \gamma - r$.

If the enslaver pursues with probability $\hat{\sigma}_1$, does a low type prefer to run or stay? Comparing S_l's utility from staying and running:

$$
\begin{aligned}
U_l(\pi=1|\hat{\sigma}_1) &> U_l(\pi=0|\hat{\sigma}_1) \\
\hat{\sigma}_1(\underline{q}\cdot w_l-(1-\underline{q})\cdot r)+(1-\hat{\sigma}_1)(\bar{q}\cdot w_l-(1-\bar{q})\cdot r) &> 0 \\
\bar{q}\cdot w_l-(1-\bar{q})\cdot r &> \hat{\sigma}_1(\bar{q}-\underline{q})(w_l+r) \\
\bar{q}(w_l+r)-r &> \frac{(w_l+r)(\gamma+r+\bar{q}(\gamma-r-w_h))}{\gamma-r-w_h} \\
\bar{q}-\frac{r}{w_l+r} &> \frac{(\gamma+r+\bar{q}(\gamma-r-w_h))}{\gamma-r-w_h} \\
-\frac{r}{w_l+r} &> \frac{\gamma+r}{\gamma-r-w_h} \\
\frac{\gamma+r}{w_h+r-\gamma} &> \frac{r}{w_l+r} \\
w_l\cdot\gamma+2\gamma\cdot r+w_l\cdot r &> r\cdot w_h \\
\frac{w_l\cdot\gamma}{r}+2\gamma+w_l &> w_h
\end{aligned}
$$

If $\frac{w_l\cdot\gamma}{r}+2\gamma+w_l > w_h > \frac{\gamma(1+\underline{q})+r(1-\underline{q})}{\underline{q}}$, then the low type prefers to run while the high type is indifferent. If, however, $w_h > \frac{w_l\cdot\gamma}{r}+2\gamma+w_l$, then the low type prefers to stay, and there is not an equilibrium in which low types run and high types are indifferent between staying or concealing and running.

Given that the enslaver is indifferent between pursuing and not, we can solve for the probability that S_h conceals, $1-\psi$. Because a high type never produces low and stays, $\pi_h(c_l)=1$. Consider the case with wage values $\frac{w_l\cdot\gamma}{r}+2\gamma+w_l > w_h > \frac{\gamma(1+\underline{q})+r(1-\underline{q})}{\underline{q}}$, such that low types prefer to run, $\pi_l^*(c_l)=1$.

$$\begin{aligned}
\mu(c_l, e_S = 1) &= \frac{k}{(\bar{q}-\underline{q})(c_h-\gamma)} \\
\frac{Pr(c_l, e_S = 1|\theta = h)\cdot\rho}{Pr(c_l, e_S = 1|\theta = h)\cdot\rho + \pi_l(c_l)\cdot(1-\rho)} &= \frac{k}{(\bar{q}-\underline{q})(c_h-\gamma)} \\
\frac{(1-\psi)\cdot\pi_h(c_l)\cdot\rho}{(1-\psi)\cdot\pi_h(c_l)\cdot\rho + 1-\rho} &= \frac{k}{(\bar{q}-\underline{q})(c_h-\gamma)} \\
(1-\psi)\cdot\rho\cdot(\bar{q}-\underline{q})(c_h-\gamma) &= ((1-\psi)\cdot\rho + 1-\rho)\cdot k \\
(1-\psi)\cdot\rho\cdot((\bar{q}-\underline{q})(c_h-\gamma)-k) &= (1-\rho)\cdot k \\
(1-\psi) &= \frac{(1-\rho)\cdot k}{\rho\cdot((\bar{q}-\underline{q})(c_h-\gamma)-k)} \\
\psi^* &= 1-\frac{(1-\rho)\cdot k}{\rho\cdot((\bar{q}-\underline{q})(c_h-\gamma)-k)}
\end{aligned}$$

The high type conceals with probability $1-\psi^* = \frac{(1-\rho)\cdot k}{\rho\cdot((\bar{q}-\underline{q})(c_h-\gamma)-k)}$, and $\psi \in (0,1)$ when $\rho(\bar{q}-\underline{q})(c_h-\gamma) > k$.

D.2.2

High types are indifferent between producing high and running versus concealing and running when the enslaver pursues with probability $\hat{\sigma}_2 = \frac{\gamma}{(\bar{q}-\underline{q})(\gamma-w_h-r)} + 1$.

$$\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 1|\sigma(c_l)\in(0,1)) &= U_h(\psi = 0, \pi(c_l) = 1|\sigma(c_l)\in(0,1)) \\
\gamma + \underline{q}\cdot w_h + (1-\underline{q})(\gamma-r) &= \sigma(\underline{q}\cdot w_h + (1-\underline{q})(\gamma-r)) + (1-\sigma)(\bar{q}\cdot w_h + (1-\bar{q})(\gamma-r)) \\
\gamma + (\bar{q}-\underline{q})(\gamma-w_h-r) &= \sigma(\bar{q}-\underline{q})(\gamma-w_h-r) \\
\frac{\gamma}{(\bar{q}-\underline{q})(\gamma-w_h-r)} + 1 &= \hat{\sigma}_2
\end{aligned}$$

$\hat{\sigma}_2 \in (0,1)$ when $w_h > \frac{\gamma}{\bar{q}-\underline{q}} + \gamma - r$. Does a low type prefer to run or stay when the enslaver pursues with probability $\hat{\sigma}_2$? Comparing S_l's utility from staying and running:

$$\begin{aligned}
U_l(\pi=1|\hat{\sigma}_2) >& \quad U_l(\pi=0|\hat{\sigma}_2) \\
\bar{q}\cdot w_l - r(1-\bar{q}) >& \quad \hat{\sigma}_2(\bar{q}-\underline{q})(w_l+r) \\
\bar{q}\cdot(w_l+r)-r >& \quad (\bar{q}-\underline{q})(w_l+r)+\frac{\gamma(w_l+r)}{\gamma-w_h-r} \\
\underline{q}\cdot(w_l+r)-r >& \quad \frac{\gamma(w_l+r)}{\gamma-w_h-r} \\
w_l(\underline{q}-\frac{\gamma}{\gamma-w_h-r}) >& \quad r(1-\underline{q})+\frac{\gamma\cdot r}{\gamma-w_h-r} \\
w_l(\underline{q}(\gamma-w_h-r)-\gamma) <& \quad (2-\underline{q})\gamma\cdot r - r(1-\underline{q})(w_h+r) \\
w_l(\underline{q}(\gamma-w_h-r)-\gamma) <& \quad r(2-\underline{q}-(1-\underline{q})(w_h+r)) \\
w_l >& \quad \frac{r(2-\underline{q}-(1-\underline{q})(w_h+r))}{\underline{q}(\gamma-w_h-r)-\gamma}
\end{aligned}$$

Rewriting in terms of w_h, we have that low types prefer to run if

$$w_h > \frac{r^2(1-\underline{q})+r(\underline{q}-2-\underline{q}\cdot w_l)-\gamma\cdot w_l(1-\underline{q})}{\underline{q}\cdot w_l - r(1-\underline{q}}$$

Recall that $\hat{\sigma}_2 \in (0,1)$ when $w_h > \frac{\gamma}{\bar{q}-\underline{q}}+\gamma-r$, and note that $\frac{\gamma}{\bar{q}-\underline{q}}+\gamma-r$ is always greater than the stated w_h cutoff. Thus, low types always run when $\sigma^* = \hat{\sigma}_2$.

As before, the high type conceals with probability $1-\psi^* = \frac{(1-\rho)\cdot k}{\rho\cdot((\bar{q}-\underline{q})(c_h-\gamma)-k)}$, and $\psi \in (0,1)$ when $\rho(\bar{q}-\underline{q})(c_h-\gamma) > k$.

The equilibrium strategy profile when $rho > \frac{k}{\bar{k}}$ follows. This mirrors the presentation in proposition 5.

For $\rho > \frac{k}{\bar{k}}$,

a) $(\psi^* = \frac{\rho\cdot\bar{k}-k}{\rho(\bar{k}-k)}$, $\pi_l^*=1$, $\pi_h^*(c_l)=1$, $\pi_h^*(c_h)=0$, $\sigma^*(c_l)=\hat{\sigma}_1$, $\sigma^*(c_h)=1$, $\mu=\hat{\mu})$ when $w_h \in (\frac{w_l\cdot\gamma}{r}+2\gamma+w_l, \frac{\gamma(1+\underline{q})+r(1-\underline{q})}{\underline{q}})$

b) $(\psi^* = \frac{\rho \cdot \bar{k} - k}{\rho(\bar{k} - k)}$, $\pi_l^* = 1$, $\pi_h^*(c_l) = 1$, $\pi_h^*(c_h) = 1$, $\sigma^*(c_l) = \hat{\sigma}_2$, $\sigma^*(c_h) = 1$, $\mu = \hat{\mu})$ when $w_h > \frac{\gamma}{\bar{q} - \underline{q}} + \gamma - r$

D.3 Suppose S_h and S_l produce low and run.

Recall that S_θ prefers to run when they are pursued with probability below a certain threshold. If both low and high types want to run, then they must be pursued with a probability less than $\min\{\sigma^*, \hat{\sigma}_1, \hat{\sigma}_2\}$.

$$
\begin{aligned}
U_h(\psi = 0, \pi(c_l) = 1 | \sigma(c_l) \in (0,1)) > & \quad \max\{U_h(\psi = 1, \pi(c_h) = 1 | \sigma(c_l) \in (0,1)), \\
& \quad U_h(\psi = 1, \pi(c_h) = 0 | \sigma(c_l) \in (0,1))\} \\
U_l(\pi = 1 | \sigma(c_l) \in (0,1)) > & \quad 0
\end{aligned}
$$

Recall that $\hat{\sigma}_1 \in (0,1)$ when $w_h > \frac{\gamma(1+\underline{q}) + r(1-\underline{q})}{\underline{q}}$. Note that when $w_h > \frac{\gamma(1+\underline{q}) + r(1-\underline{q})}{\underline{q}}$, $w_h > \gamma - r$. And $\hat{\sigma}_2 \in (0,1)$ when $w_h > \frac{\gamma}{\bar{q} - \underline{q}} + \gamma - r$.

$$
\begin{aligned}
\hat{\sigma}_1 < & \quad \hat{\sigma}_2 \\
\frac{\gamma + r + \bar{q}(\gamma - r - w_h)}{(\bar{q} - \underline{q})(\gamma - r - w_h)} < & \quad \frac{\gamma}{(\bar{q} - \underline{q})(\gamma - w_h - r)} + 1 \\
\gamma + r + \bar{q}(\gamma - r - w_h) > & \quad \gamma + (\bar{q} - \underline{q})(\gamma - r - w_h) \\
r > & \quad -\underline{q}(\gamma - r - w_h) \\
r(1 - \underline{q}) > & \quad \underline{q}(w_h - \gamma) \\
\frac{r(1 - \underline{q})}{\underline{q}} + \gamma > & \quad w_h
\end{aligned}
$$

Comparing this to the conditions above, we see that $\hat{\sigma}_2 \ngeq \hat{\sigma}_1$. Therefore we want to compare $\hat{\sigma}_2$ to σ^*.

$$\begin{aligned}
\sigma^* &< \hat{\sigma}_2 \\
\frac{\bar{q}\cdot w_l-(1-\bar{q})r}{(\bar{q}-\underline{q})(w_l+r)} &< \frac{\gamma}{(\bar{q}-\underline{q})(\gamma-w_h-r)}+1 \\
(\bar{q}\cdot w_l-(1-\bar{q})r)(\gamma-w_h-r) &> (\gamma+\bar{q}-\underline{q})(w_l+r)
\end{aligned}$$

The left-hand side is negative, given the conditions under which $\sigma^* \in (0,1)$ and $\hat{\sigma}_2 \in (0,1)$. The right-hand side is positive, and so $\sigma^* \not\leq \hat{\sigma}_2$. Thus, $\hat{\sigma}_2 = \min\{\sigma^*, \hat{\sigma}_1, \hat{\sigma}_2\}$. As such, both types want to run when they are pursued with probability $\sigma^\dagger := \sigma < \hat{\sigma}_2$. Returning to the conditions on $\hat{\sigma}_2$, $\sigma^\dagger \in (0,\hat{\sigma}_2)$ when $w_h > \frac{\gamma}{\bar{q}-\underline{q}} + \gamma - r$.

The equilibrium strategy profile when $\rho = \frac{k}{\bar{k}}$ follows. This mirrors the presentation in proposition 6.

For $\rho > \frac{k}{\bar{k}}$,

$(\psi^* = 0,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \sigma^*(c_l) = \sigma^\dagger,\ \sigma^*(c_h) = 1,\ \mu = \rho)$ when $w_h > \frac{\gamma}{\bar{q}-\underline{q}} + \gamma - r$